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Documentation of the PUblic Policy Model for Austria and other European countries (PUMA)

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1 Introduction and Basic Structure of the Model

This Research Paper contains a detailed documentation of the dynamic computable general equilibrium model PuMA. The model is used for analysing economic, labour market and public finance effects of different policy options, structural changes and other important policy questions. PuMA is similar to the EU Labour Market Model (EU-LMM), which was also developed by the authors and is used by the Directorate General Employment, Social Affairs & Inclusion of the European Commission[1].

The *demographic structure* of PuMA is based on the probabilistic ageing (PA) approach introduced by Grafenhofer et al. (2007) which allows for a period length of one year and approximates life-cycle features with a limited number of age groups to keep the dimensions of the model more tractable. The PA model is a generalization of Gertler (1999) who was the first to introduce a simple life-cycle structure into the basic Blanchard (1985) perpetual youth model. Keuschnigg and Keuschnigg (2004) combine dynamic search unemployment with the Gertler model and model two age groups, workers and retirees, to analyse pension reforms. The PA model and PuMA consider several age groups of workers and retirees and allow for mortality in all life-cycle stages. They are, thus, Overlapping Generations (OLG) Models in the spirit of Samuelson (1958) and Diamond (1965). Different generations sharing the same age characteristics are aggregated analytically to a low number of eight age groups. As a result, life-cycle profiles of labour productivity, income or unemployment rates are replicated in sufficient detail. The PA model thus allows for a very parsimonious yet rather accurate approximation of demographic change and of life-cycle differences in earnings, wealth and consumption.[2] The model includes a detailed representation of public social insurance systems in the presence of life-cycle labour supply, training, search unemployment and retirement. In addition, migration of households is possible[3].

A main focus of the model lies in a detailed analysis of household behaviour. We differentiate labour force with respect to both age and skills, so that we are able to highlight substitutions and complementarities between the different skills, and between labour and capital. In particular, we use a CES-production function with three nests that captures the feature of capital-skill complementarity (see Griliches (1969), Goldin and Katz (1998), Krusell et al. (2000)). Empirical evidence suggests that high skilled labour and capital are more complementary than unskilled labour and capital. We assume that, independent of age and gender, labour input of the same skill group is perfectly substitutable. Labour productivity of agents is age-dependent, influenced by endogenous life-long learning and firm-sponsored training decisions, and, together with labour market status and institutions, determines the life-cycle earnings profile.

[1]See e.g. Berger et al. (2016) and European Commission (2017) for applications.

[2]An alternative approach is to rely on models with a very large number of generations in the style of Auerbach and Kotlikoff (1987). Life-cycle models with many generations can take account of detailed differences in wealth, marginal propensity to consume and in labour supply and earnings of different agents. But precisely for this reason, they are computationally expensive to implement. This problem becomes particularly pronounced if the model contains a considerable amount of heterogeneity (several skill groups and several different labour market states) and extensive institutional details. For this reason, a model of the Auerbach-Kotlikoff-type is less tractable.

[3]In the basic version of PuMA, described in this document, we assume that migrants share the same labour market characteristics in a particular demographic group. In an extended version described in the Appendix, divergent characteristics are possible. In both versions, migration is an exogenously made decision.

Firms are divided in two different types, following Keuschnigg and Kohler (2002). *Final goods firms* produce output by combining four types of inputs (capital and low-, medium- and high-skilled labour). Production of final goods firms satisfies demand (private and public consumption, investment and export demand) and they set prices by which the goods are supplied. They operate in a monopolistic competition environment. The capital stock necessary for production is rented from *Capital goods firms* for which they pay a rental price. The market for capital goods is characterized by perfect competition. Capital goods firms buy goods from final goods firms and transform them into the required capital stock. They maximize the present value of profits by optimally choosing investment according to the q-theory of Hayashi (1982). Investment is associated with installation costs (Barro and Sala-i-Martin (1995)).

Final goods firms maximize profits by choosing the number of vacancies, the share of workers they fire, the amount of firm-sponsored training, the level of rented capital stock and capital utilization (see e.g. Ratto et al. (2009)). To take into account for more pronounced short term frictions in labour adjustment, the process of adjustment leads to additional costs (see Ejarque and Portugal (2007)). In general, labour market frictions are the result of a search unemployment setting (static version) as pioneered by Mortensen (1986). A separate matching function exists for each of the age- and skill groups. However, there are considerable interactions among the different skill groups, mainly caused by the CES-production function. Labour demand for the different age- and skill groups depends on labour productivity, labour costs, labour market tightness and institutional details.

Population is distinguished with respect to *age and education.* The life-cycle is divided into periods of education, active work and retirement. At the beginning of their lifetime, individuals endogenously decide to which skill group they belong. In this respect, we distinguish three different groups, low (ISCED 0-2), medium (ISCED 3-4), and high skilled (ISCED 5+). As the acquisition of education requires time, more highly educated individuals enter the labour force at a higher age. *Lifelong learning* enhances labour productivity. However, training doesn't allow individuals to switch into the next higher skill group. Each individual has a stock of available productivity (human capital). Investment in lifelong learning increases this stock, but the stock also depreciates in each period. Members of the group of older workers endogenously choose an optimal retirement age, so that a share of this group is still working while the other share is already retired.

The *labour market status* of an individual can be employed, unemployed, inactive or retired. Household income consists of (net) labour income, unemployment or pension benefits, severance payments from the firms, transfers from the government, inter-vivo transfers from other households and capital income. Aggregation makes use of the concept of income-pooling elaborated by Andolfatto (1996). Savings follow from an optimal *intertemporal consumption choice* with perfect foresight. Households save to ensure smooth consumption in the face of a fluctuating life-cycle income pattern. In particular, they save to top up pension benefits and sustain their consumption level during retirement. We allow for intergenerational transfers which are important in replicating consumption profiles.

Individuals maximize their lifetime utility by endogenously choosing the number of hours worked if employed, the search intensity for a job if unemployed, participation in the labour market, and the retirement age. They also choose educational investment at the beginning of their lifetime (age 15), the effort invested in lifelong learning activities and an optimal intertemporal allocation of consumption. Sections 3 describes in detail the optimization decisions of households.

The model contains *search unemployment* as pioneered by Mortensen (1986). Final goods firms post an optimal number of vacancies and unemployed persons choose an optimal search effort. Vacancies and individuals are allocated by job matching. We use a static search model as in Boone and Bovenberg (2002) in contrast to a dynamic model as in Pissarides (2000). The static model is simpler yet it captures the essential insights of the dynamic one. Subsequently, final goods firms endogenously choose an optimal share of the work force they want to keep within the firm. This share is determined by labour productivity, wage costs, employment protection legislation and managerial costs to keep a worker. Furthermore, firms decide on the optimal amount of firm-sponsored training, which is determined by the increase of labour productivity and the costs associated with training. In combination with private training and education, firm-sponsored training determines the productivity of an employee. Given the bargaining power of workers and firms, employers and employees bargain over wages. In the simplest Nash-bargaining setting, they take into account labour productivity and the outside option of the employee, which mainly depends on the unemployment replacement rate and the value of home production. In addition, institutional settings are taken into account, e.g. the tax and benefit system. Due to the structure of the model, we can reflect age- and skill-dependent unemployment rates.

Returns to education are influenced by skill-dependent wage differentials as well as endogenously determined employment prospects and institutional arrangements like social protection and progressive income taxation. The coexistence of endogenously determined productivity formation and a search model with labour market frictions is therefore an important feature of the model, as individuals consider the employment prospects for their educational decisions. By changing the employment prospects of individuals, policy changes influence the educational decisions. This is different to well-known models used by Heckman, Lochner and Taber (1998a, 1998b, 1999), Blundell, Costa Dias and Meghir (2003), Laxton et al. (1998) or Altig et al. (2001) which do not include labour market imperfections. Therefore, PuMA can capture the effects of labour market frictions on the incentives for education.

The model includes a detailed description of the *public sector* and *institutional settings*. The budget of the public sector is divided into a budget for *social insurance* and a *general* budget. Expenditures for social insurance are financed by social security contributions of employers and employees and transfers from the general budget. Even if pensions and unemployment benefits are linked to previous earnings, contributions to a PAYG system are partly perceived as taxes if they earn a rate of return less than the market interest rate. These implicit taxes are part of the overall tax burden on labour and importantly determine the labour market outcome as they can be a large part of the total tax wedge (much as in Fisher and Keuschnigg (2007), who characterize the effective tax rates of social security contributions that distort labour market behaviour of prime age workers and endogenous retirement of older workers). Revenues of the general budget comprise all main taxes, e.g. personal income tax, corporate income tax, consumption taxes, capital and capital gains taxes. Public expenditures include government consumption, transfers to the social security systems and to households, subsidies to firms and debt servicing. For each of the two budgets mentioned above, different rules for budget closure can be set. To follow these rules on the revenue side, corresponding social security rates as well as different tax rates can be adjusted. It is also possible to set rules for the expenditure side of these budgets. These rules can be set in a flexible way in the model.

The countries considered in PuMA are modelled as *small open economies* in the sense that the interest rate is internationally determined. Saving-investment imbalances lead to an endogenously determined net foreign asset position which is reflected in a trade balance surplus or deficit. We assume that, prior to policy changes, the economy follows a balanced growth path. After a policy reform is implemented, it enters a prolonged period of transition and finally converges to a new balanced growth path. The model thus provides transitory as well as long-run effects of policy reforms.

Simulation results include macroeconomic effects (e.g. on GDP, investment, consumption) and labour market effects (e.g. unemployment and employment rates, number of hours worked, wages and average productivity) of reforms. Household specific variables can be presented in an aggregate manner for the entire economy, and also separately for each of the groups or partially aggregated (i.e. age- or skill-dependent). Based on the model we can analyze inter- as well as intragenerational, and intertemporal effects of reforms.

The following model documentation is structured as follows. Section 2 explains the life-cycle structure, section 3 the household sector of the economy, section 4 the demand for varieties of goods and section 6 the production side. In section 7, we describe the public sector. Section 8 describes the macroeconomic equilibrium. The description of a welfare measure can be found in section 9. Functional forms used can be found in section 10 and section 11 describes some calibration features of the model. The two Appendices (section 12 and section 13) provide information on the detailed migration model extension and proofs.

2 Life-Cycle Description of Households

This section provides a brief overview of the demographic structure of the model. For a more refined description, see Grafenhofer et al. (2007), who introduced the concept of 'Probabilistic Ageing' (PA). We define a discrete number of A states of increasing age, and accordingly collect all agents with identical characteristics in the same age group $a \in \{1, \ldots, A\}$. People start life in state $a = 1$ with the age of 15 with a given set of attributes. The life-cycle characteristics include a person's earnings potential and her mortality risk but other attributes as well. Ageing means that an individual's life-cycle characteristics change when she grows older, i.e. switches to state $a+1$. We measure real time in regular annual periods. The ageing clock runs slower and stochastically. In PuMA, we define $A = 8$ different age groups.

Households differ not only by their date of birth, but also by their diverse life-cycle histories. An agent's life-cycle history is her biography of ageing events that have happened since birth. It is represented by a vector α that records the past dates of ageing events. At date t, the set of possible histories of a household that belongs to age group a is

$$\mathcal{N}_t^a \equiv \{(\alpha_1, \ldots, \alpha_a) : \alpha_1 < \ldots < \alpha_a \leq t\}. \tag{1}$$

A particular life-cycle history is represented by a vector $\alpha \in \mathcal{N}_t^a$. The element α_i, $i \in \{1, \ldots, a\}$, denotes the period at which the household who was formerly in age group $i-1$, became a member of group i. Individual biographies are updated when a person experiences an ageing event. Suppose a person is in age group $a-1$ and is identified by a biography $\alpha = (\alpha_1, \ldots, \alpha_{a-1})$. If an ageing shock occurs at the end of period t, she changes to age group a at the beginning of the next period. Her biography is appended by the entry $t+1$ and reads $(\alpha_1, \ldots, \alpha_{a-1}, t+1)$.

To model demographics, we allow for mortality among younger age groups. When an individual with an arbitrarily given life-cycle history plans for following periods, she faces the risk of ageing and dying. In a particular age group, agents are identical and face the same independent probability of moving to one of the alternative states. She must thus reckon with three possible events: (i) she dies with probability $1-\gamma^a$; (ii) she survives without ageing and remains in the same age group with probability $\gamma^a\omega^a$, and (iii) she survives, ages and belongs to age group $a+1$ in the next period with probability $\gamma^a(1-\omega^a)$. With stochastically independent risks, the law of large numbers implies that the individual probabilities for a certain event correspond to the fraction of people that are subject to this event. The number of agents at date t, in state of life a and with history α is given by $N_{\alpha,t}^a$. The transition process is thus given by

$$\begin{array}{llll} (i) & N^{\dagger}_{\alpha^{\dagger},t+1} & = & N^a_{\alpha,t} \cdot (1-\gamma^a), & \text{death}, \\ (ii) & N^a_{\alpha,t+1} & = & N^a_{\alpha,t} \cdot \gamma^a \omega^a, & \text{no ageing}, \\ (iii) & N^{a+1}_{\alpha',t+1} & = & N^a_{\alpha,t} \cdot \gamma^a (1-\omega^a), & \text{ageing}. \end{array} \tag{2}$$

Individuals in the last age group have exhausted the ageing process, implying $\omega^A = 1$ as an end condition. They may either survive with probability γ^A within group A or die with probability $1-\gamma^A$. Observe that only the last age group behaves according to the mortality and demographic assumptions in Blanchard's (1985) perpetual youth model.

The total number of people in age group a is obtained by summing up over all possible histories α ending up in this age group:

$$N_t^a \equiv \sum_{\alpha \in \mathcal{N}_t^a} N_{\alpha,t}^a. \tag{3}$$

Migration In addition to the PA structure, we allow for migration in PuMA. At the beginning of a period and for each group, a number of *netmig* net migrants enter the modelled country ($netmig < 0$ indicating net outward migration). In the basic version of PuMA described here, we assume that migrants share average labour market characteristics (in a particular demographic group).[4] Furthermore, they transfer individual stocks of assets, pension claims and productivity when migrating (which are assumed to be a fraction of stocks of native people; for details see later).

The key demographic parameters are the birth rate, transition rates to successive age groups, mortality rates and the number of net migrants. The law of motion for age groups is therefore given by:

[4] An extended version which is described in the Appendix, allows for different characteristics of natives and migrants.

Proposition 1 ***(Demographic Structure)***

$$N_{t+1}^{a} = \gamma^{a}\omega^{a}N_{t}^{a} + \gamma^{a-1}\left(1-\omega^{a-1}\right)N_{t}^{a-1} + netmig_{t+1}^{a}, \quad \omega^{A}=1, \tag{4}$$

$$N_{t+1}^{1} = \gamma^{1}\omega^{1}N_{t}^{1} + N_{(t+1),t+1}^{1} + netmig_{t+1}^{1}, \tag{5}$$

$$N_{t+1} = N_{t} + N_{(t+1),t+1}^{1} - \sum_{a=1}^{A}\left(1-\gamma^{a}\right)N_{t}^{a} + netmig_{t+1}, \quad N_{t} \equiv \sum_{a=1}^{A} N_{t}^{a}, \tag{6}$$

where $N_{(t+1),t+1}^{1}$ is the mass of newborns in period $t+1$ and $netmig = \sum_{a=1}^{A} netmig^{a}$.

Proof. For the proof, see Grafenhofer et al. (2007). ■

Remark 1 *Note, that other well-known OLG models, like Auerbach Kotlikoff (1987), Blanchard- (1985) and Gertler-(1999) models can be replicated as special cases of the PA model by setting the parameters γ and ω in appropriate ways.*

Remark 2 *A detailed understanding of the demographic PA process of the model is not entirely essential for an understanding of the other parts of the PuMA model. It is sufficient to remember that in each period an individual faces two types of 'risk': ageing (i.e. changing into the next age group) $(1-\omega^{a})$ and dying $(1-\gamma^{a})$ in each period.*

3 Private Households Optimization

Households are distinguished with respect to both their age and educational level. The life-cycle starts at age 15, when individuals endogenously decide which educational group ($i = 1, 2$ or 3 for low-, medium- and high-skilled) to join. This extensive educational decision will be discussed later (see subsection 3.3). For the moment, we consider this choice as given. Individuals with the lowest level of education start working immediately. The other two groups stay in school for a longer period, and they finance their consumption of goods via debt and transfers from older households and the government. The second group starts working at an age of 20 and the third group at age 25. After the initial educational decision, switching to other skill groups is impossible. In total, we model $A = 8$ age groups: three purely retired groups, one mixed groups (of workers and retirees, see below) and four working age groups.

We first describe the intertemporal optimization problem of retirees and subsequently that of working groups.

3.1 Retirees

The last three age groups ($a = 6, 7, 8$) consist of retirees. The households belonging to the oldest working group ($a = 5$) decide endogenously about their retirement age. This group is treated analogously to the worker groups and is therefore described in the next section. Retirees receive pension payments from the public pension system. Their consumption is partly financed from previously accumulated financial assets. Public *funded* pension systems could be seen as perfect substitutes for private savings of households. Without credit constraints, higher statutory payments to public funded systems will increase savings held in funded system and one-by-one lower private saving if the rate of return of these two types of savings is the same. However, we include public funded pension systems in the model as the tax treatment of contributions and benefits may differ from the tax treatment of private savings.

Retiree households maximize their present value of utility (value function) by choosing an optimal amount of consumption in each period. Preferences are given by a CES expected utility function[5], see Farmer (1990) and Weil (1990). In period t, a retired person with biography α, age a and skill i solves

$$\begin{aligned} V\left(A^a_{\alpha,i,t}, P^a_{\alpha,i,t}\right) &= \max_{C^a_{\alpha,i,t}} \left[\left(C^a_{\alpha,i,t}\right)^\rho + \gamma^a \beta \left(G\bar{V}^a_{\alpha,i,t+1}\right)^\rho\right]^{1/\rho}, \\ \bar{V}^a_{\alpha,i,t+1} &= \omega^a V^a_{\alpha,i,t+1} + \left(1-\omega^a\right) V^{a+1}_{\alpha',i,t+1}. \end{aligned} \tag{7}$$

Given the stock of financial assets $A^a_{\alpha,i,t}$ and pension rights $P^a_{\alpha,i,t}$, the household chooses an optimal amount of consumption (the control variable) at time t, $C^a_{\alpha,i,t}$.

Remark 3 *In order to simplify notation, we introduce the following two conventions. First, most equations of the model are the same for all skill groups. Thus, from now on, we suppress the index i for the skill groups in the description of the household sector. Second, we suppress the time index of variables whenever there is no intertemporal relation within an equation.*

[5]There are two sources of uncertainty, ageing and dying.

The constant elasticity of intertemporal substitution is given by $\sigma = 1/(1-\rho)$. The discount factor is given by $\gamma^a\beta$, where the subjective discount factor β represents the impatience of the household and γ^a the probability of surviving to the next period. A person's expected utility next period, conditional on surviving, is $\bar{V}^a_{\alpha,t+1}$. With probability ω^a, the agent remains in the same age group and expects welfare $V^a_{\alpha,t+1}$ next period. With probability $1-\omega^a$, she ages and expects welfare $V^{a+1}_{\alpha',t+1}$. In this case, the agent's biography must be updated from α to α'. Retiree groups $a < A$ are subject to an ageing risk while the last group A is not ($\omega^A = 1$), leading to $\bar{V}^A_{\alpha,t+1} \equiv V^A_{\alpha,t+1}$.

Growth is labour-augmented. In the long run, labour productivity grows with an exogenous growth trend g. The gross growth rate is therefore given by $G = (1+g)$. We calculate changes along a balanced growth path and therefore need to detrend the model, i.e. the results are normalized with respect to the raising productivity of the labour input. Furthermore, PuMA is 'real' in the sense that we detrend inflation.

Pension rights (points) are accumulated during active life by paying social security contributions to the public pension system. For retirees, stocks of pension rights evolve according to:

$$GP^a_{\alpha,t+1} = R^{P,a}P^a_{\alpha,t}. \tag{8}$$

The growth of pension entitlements includes a policy choice with respect to benefit indexation (factor $R^{P,a} = 1 + r^{P,a}$). With full wage indexation ($R^{P,a} = G$) pensions after retirement grow in line with wages in the steady state. With price indexation ($R^{P,a} = 1$ and $r^{P,a} = 0$) benefits remain constant in real terms so that pension benefits grow slower than wage earnings during the retirement period and living standard of pensioners relative to prime age workers is eroded.[6]

Retirees receive net pension income:

$$y^a_\alpha = \left(1-\tau^{p,a}\right)\left(\nu^a P^a_\alpha + P^a_0\right). \tag{9}$$

The stock of pension points P^a_α determines the earnings related part of the gross pension of the retiree household. In addition, households may also receive a flat pension P^a_0. The factor ν^a scales earnings related pension payments. This factor may be used to balance the social security sector via the adjustment of pension payments to secure sustainability of the system, e.g. in case of demographic ageing.

Pension income is taxed so that:

$$\left(1-\tau^{P,a}\right) = \left(1 - t^{w,P,a}\left(1 - x^{P,a}t^{ssc,P,a}\right) - t^{ssc,P,a}\right),$$

where $t^{w,P,a}$ is the average income tax rate, and $t^{ssc,P,a}$ the social security contribution rate. $x^{P,a}$ determines the amount to which social security contributions are deductible for income taxation. With $x^{P,a} = 1$, social security contributions are fully deductible, whereas they are not deductible if $x^{P,a} = 0$.

Because disability pensions play an important role in some of the countries, we also include them in the model. We thus allow the factor ν^a to depend on the type of pension: $\nu^a_{1,t}$ for the regular old age

[6]When nominal pensions $\tilde{P}_t$ grow annually with the factor G^P, we have $\tilde{P}_{t+1} = G^P\tilde{P}_t$. Dividing by productivity units which grow by $X_{t+1} = GX_t$, and noting $\tilde{P}_t = X_tP_t$, we get $GP_{t+1} = G^PP_t$ as in (8), where P is pension per unit of productivity.

pension claims and $\nu^a_{2,t}$ for scaling the disability pension claims. We express the earnings related part of disability pension benefits as a fixed share of the regular claims, which is also included in ν^a. Gross pension of a household[7] is therefore derived by multiplying P^a_α by the factor ν^a,

$$\nu^a = \bar{\delta}^a \nu^a_1 + \left(1 - \bar{\delta}^a\right) \nu^a_2.$$

We denote the share of persons eligible for disability pensions by $\left(1 - \bar{\delta}^a_t\right)$.

The intertemporal budget constraint of a retiree household is given by

$$G\gamma^a A^a_{\alpha,t+1} = R^\tau_{t+1} \left[A^a_{\alpha,t} + y^a_{\alpha,t} + z^a_t + iv^a_t - pc^a_t C^a_{\alpha,t}\right], \tag{10}$$

where

$$R^\tau_{t+1} = \left(1 + \left(1 - t^{cg}\right) r_{t+1}\right)$$

is the interest rate after capital gains taxes t^{cg}. We assume, that different forms of assets of the households (government bonds, domestic and foreign assets) pay the same interest rate, i.e. they are perfect substitutes to each other. Furthermore the interest rate is given by the world interest rate, which means that the economy is a price taker in the asset market[8]. The variable iv^a stands for net inter-vivo transfers given from other households. In calibrating these transfers we replicate the consumption life-cycle pattern. The consumption price pc^a includes taxes on private consumption such as value added tax and excise taxes. The model also includes lump-sum transfers (or taxes) z^a_t from the government. We assume the existence of reverse life-insurance contracts as in the OLG literature based on Blanchard (1985), so that the survival probability γ^a enters the budget constraint.

3.1.1 Optimization

A retired person maximizes the value function (7) s.t. the accumulation equation for pension points (8) and asset accumulation equation (10). The retiree thereby chooses an optimal intertemporal allocation of consumption. To solve for the optimal consumption policy, it is useful to define the following shadow prices. A shadow price can be interpreted as the additional present value of utility of an increase in a stock variable by an additional unit.

$$\begin{aligned}
\eta^a_{\alpha,t} &\equiv \frac{dV^a_{\alpha,t}}{dA^a_{\alpha,t}} \left(V^a_{\alpha,t}\right)^{\rho-1}, \qquad \lambda^a_{\alpha,t} \equiv \frac{dV^a_{\alpha,t}}{dP^a_{\alpha,t}} \left(V^a_{\alpha,t}\right)^{\rho-1}, \\
\bar{\eta}^a_{\alpha,t+1} &\equiv \left[\omega^a \frac{dV^a_{\alpha,t+1}}{dA^a_{\alpha,t+1}} + \left(1-\omega^a\right) \frac{dV^{a+1}_{\alpha',t+1}}{dA^{a+1}_{\alpha',t+1}}\right] \left(\bar{V}^a_{\alpha,t+1}\right)^{\rho-1}, \\
\bar{\lambda}^a_{\alpha,t+1} &\equiv \left[\omega^a \frac{dV^a_{\alpha,t+1}}{dP^a_{\alpha,t+1}} + \left(1-\omega^a\right) \frac{dV^{a+1}_{\alpha',t+1}}{dP^{a+1}_{\alpha',t+1}}\right] \left(\bar{V}^a_{\alpha,t+1}\right)^{\rho-1},
\end{aligned} \tag{11}$$

where $\bar{\eta}^a_{\alpha,t+1}$ and $\bar{\lambda}^a_{\alpha,t+1}$ are expected shadow prices of the asset stock and pension claims in period $t+1$ when agents face the risk of ageing. Optimality and envelope conditions for $C^a_{\alpha,t}$, $A^a_{\alpha,t}$, and $P^{j,a}_{\alpha,t}$ are:

$$C : \left(C^a_{\alpha,t}\right)^{\rho-1} = \beta R^\tau_{t+1} \bar{\eta}^a_{\alpha,t+1} G^{\rho-1} pc^a_t \tag{12}$$

[7] Consisting of old age and disability pensions.

[8] This assumption is debated in the literature under the term 'Feldstein-Horioka puzzle' (1980). Coakley et al. (2004) show that this puzzle does not hold in the period from 1980 to 2000 when heterogeneity and cross sectional dependence are accounted for. Although the debate is going on, we assume full flexibility on the capital market for simplicity.

and

$$A \quad : \quad \eta^a_{\alpha,t} = \beta R^\tau_{t+1} \bar{\eta}^a_{\alpha,t+1} G^{\rho-1}, \tag{13a}$$

$$P \quad : \quad \lambda^a_{\alpha,t} = \beta G^{\rho-1} \left(\gamma^a \bar{\lambda}^a_{\alpha,t+1} R^{P,a} + R^\tau_{t+1} \bar{\eta}^a_{\alpha,t+1} \nu^a_t \left(1 - \tau^{p,a}\right) \right). \tag{13b}$$

We define the term Ω^a which can be related to the marginal rate of substitution (MRS) across age groups,

$$\Omega^a_{\alpha,t+1} \equiv \omega^a + (1-\omega^a) \left(\Lambda^a_{\alpha,t+1}\right)^{1-\rho},$$

$$\Lambda^a_{\alpha,t+1} \equiv \frac{V^{a+1}_{\alpha',t+1}/C^{a+1}_{\alpha',t+1}}{V^a_{\alpha,t+1}/C^a_{\alpha,t+1}} \left(\frac{1/pc^{a+1}_{t+1}}{1/pc^a_{t+1}} \right)^{1/(1-\rho)}.$$

Lemma 1 ***(Shadow Prices)*** *Shadow prices are entirely forward looking and independent of history α, $\eta^a_{\alpha,t} = \eta^a_t$, $\lambda^a_{\alpha,t} = \lambda^a_t$. The ratio of shadow prices $\tilde{\lambda}^a_t \equiv \lambda^a_t / \eta^a_t$ satisfies*

$$\bar{\tilde{\lambda}}^a_t \equiv \omega^a \tilde{\lambda}^a_t + (1-\omega^a) \left(\Lambda^a_t\right)^{1-\rho} \tilde{\lambda}^{a+1}_t \Rightarrow \frac{\bar{\lambda}^a_t}{\bar{\eta}^a_t} = \frac{\bar{\tilde{\lambda}}^a_t}{\Omega^a_t}. \tag{14}$$

Furthermore, it can be shown that $\Omega^a_{\alpha,t} = \Omega^a_t$ and $\Lambda^a_{\alpha,t} = \Lambda^a_t$.

Remark 4 *This feature is very important for the aggregation.*

Proposition 2 ***(Euler equation for retirees)*** *The modified Euler equation for the retirees which determines the optimal intertemporal allocation of consumption is given by*

$$C^a_{\alpha,t} \left(pc^a_t \beta R^\tau_{t+1} \Omega^a_{t+1}\right)^\sigma = \omega^a \left(pc^a_{t+1}\right)^\sigma G C^a_{\alpha,t+1} + (1-\omega^a) \left(pc^{a+1}_{t+1}\right)^\sigma G C^{a+1}_{\alpha',t+1} \Lambda^a_{t+1}. \tag{15}$$

Proof. see Appendix. ■

According to the modified Euler equation the optimal consumption profile equates the marginal utility of consumption in each period to the expected discounted marginal utility of postponing consumption to the next period.

Proposition 3 ***(Retiree Policy)***

$$pc^a C^a_\alpha = \frac{1}{\Delta^a} \left(A^a_\alpha + S^a_\alpha + T^a_\alpha\right), \tag{16}$$

$$V^a_\alpha = \left(\Delta^a\right)^{1/\rho} C^a_\alpha, \tag{17}$$

where

$$\Delta^a_t = 1 + \left(\frac{pc^a_t}{pc^a_{t+1}}\right)^{\sigma-1} (\beta)^\sigma \left(R^\tau_{t+1} \Omega^a_{t+1}\right)^{\sigma-1} \gamma^a \Delta^a_{t+1}, \tag{18}$$

$$T^a_{\alpha,t} = z^a_t + iv^a_t + \left(1-\tau^{p,a}\right) P^a_0 + \frac{\gamma^a}{R^\tau_{t+1} \Omega^a_{t+1}} G \bar{T}^a_{\alpha,t+1}, \tag{19}$$

$$S^a_{\alpha,t} = \left(1-\tau^{p,a}\right) P^a_{\alpha,t} \nu^a_t + \frac{\gamma^a}{R^\tau_{t+1} \Omega^a_{t+1}} G \bar{S}^a_{\alpha,t+1}, \tag{20}$$

$$\bar{T}^a_{\alpha,t+1} = \omega^a T^a_{\alpha,t+1} + (1-\omega^a) T^{a+1}_{\alpha',t+1} \left(\Lambda^a_{t+1}\right)^{1-\rho},$$

$$\bar{S}^a_{\alpha,t+1} = \omega^a S^a_{\alpha,t+1} + (1-\omega^a) S^{a+1}_{\alpha',t+1} \left(\Lambda^a_{t+1}\right)^{1-\rho}.$$

The marginal propensity to consume is $1/\Delta^a_t$ and S and T are pension and transfer wealth. End conditions for the last age group are $\omega^A = \Omega^A_{t+1} = 1$, $\bar{S}^{j,A}_{\alpha,t+1} = S^{j,A}_{\alpha,t+1}$ and $\bar{T}^A_{\alpha,t+1} = T^A_{\alpha,t+1}$.

Proof. See the Appendix for the proof of 16 and 17. ∎

According to proposition 3, a retiree's optimal policy is to spend in each period a fraction of her wealth on current consumption C. Wealth consists of her previously accumulated financial assets A plus pension wealth S (the present value of future, earnings related, pension entitlements) plus transfer wealth T. Transfer wealth T includes the present value of future flat pensions, lump-sum transfers and inter-vivo transfers. Reflecting mortality risk, older people have a higher marginal propensity to consume out of life-time wealth than younger ones, i.e. they consume a higher share of their wealth. V_{α}^{a}, the present value of utility of an individual of age group a with biography α, is given by $(\Delta^{a})^{1/\rho}\, C_{\alpha}^{a}$.

Box: Consumption and Inter-Vivo Transfers Consumption C_{α}^{a} consists of two parts in PuMA, the private consumption of goods c_{α}^{a} and inter-vivo transfers tr_{α}^{a} to other households. These transfers tr_{α}^{a} are the counterpart to the transfers received iv_{t}^{a}. These inter-vivo transfers are integrated in the model in order to replicate an age-profile of private consumption of goods that corresponds to the data. (For a similar reason, Taber (2002) introduces transfers from the old to the young.) We assume so-called 'egoistic preferences' which means that households gain utility from *giving transfers* (and not from increasing utility of other households). To keep things simple, we assume that the utility function with respect to this consumption bundle is a (linearly homogenous and strictly quasiconcave) CES-function, which allows to seperate the maximization problem with respect to first the intertemporal allocation of private consumption C_{α}^{a} and second the intratemporal division into consumption goods c_{α}^{a} and transfers tr_{α}^{a}.

The **expenditure minimization problem** can be expressed by:

$$\min_{c_{\alpha,t}^{a},tr_{\alpha,t}^{a}} pc^{a}\left(c_{\alpha,t}^{a}+tr_{\alpha,t}^{a}\right) \qquad s.t. \tag{21}$$

$$\left[\left(a_{c}^{a}\right)^{\frac{1}{\sigma_c}}\left(c_{\alpha,t}^{a}\right)^{\frac{\sigma_c-1}{\sigma_c}}+\left(1-a_{c}^{a}\right)^{\frac{1}{\sigma_c}}\left(tr_{\alpha,t}^{a}\right)^{\frac{\sigma_c-1}{\sigma_c}}\right]^{\frac{\sigma_c}{\sigma_c-1}} \geq \bar{U}, \tag{22}$$

where a_{c}^{a} is the utility weight on consumption goods and σ_c the elasticity of substitution. Standard linear optimization methods reveal the optimal division of this consumption bundle into consumption of goods and transfers.

Proposition 4 ***(Division of Consumption Bundle)*** *Given the optimal amount of consumption $C_{\alpha,t}^{a}$, the optimal level of consumption and inter vivo transfers is given by:*

$$\begin{aligned} pc^{a}c_{\alpha,t}^{a} &= a_{c}^{a}pc^{a}C_{\alpha,t}^{a}, \\ pc^{a}tr_{\alpha,t}^{a} &= \left(1-a_{c}^{a}\right)pc^{a}C_{\alpha,t}^{a}. \end{aligned} \tag{23}$$

Proof. see Appendix. ∎

Remark 5 *This division of consumption $C_{\alpha,t}^{a}$ also holds for workers in section 3.2.*

It can be shown that pension wealth, consisting of the present value of earnings related pensions, is the product of the stock of pension entitlements and its shadow price. Note, however, that earnings related pension wealth $S^a_{\alpha,t}$ is only part of total pension wealth, while the present value of lump-sum pensions P^0_t is included in $T^a_{\alpha,t}$.

Proposition 5 ***(Earnings Related Pension Wealth of Retirees)*** *Earnings related pension wealth is*

$$S^a_{\alpha,t} = \tilde{\lambda}^a_t P^a_{\alpha,t}, \tag{24}$$

where $\tilde{\lambda}^a_t \equiv \lambda^a_t/\eta^a_t$. *Furthermore, the shadow price of the pension stock evolves according to*

$$\tilde{\lambda}^a_t = \left(1 - \tau^{p,a}\right)\nu^a_t + \frac{\gamma^a R^{P,a}}{R^\tau_{t+1}\Omega^a_{t+1}}\left[\omega^a \tilde{\lambda}^a_{t+1} + \left(1-\omega^a\right)\tilde{\lambda}^{a+1}_{t+1}\left(\Lambda^a\right)^{1-\rho}\right].$$

Proof. see Appendix. ■

3.2 Workers and Mixed Group

This section explains the behaviour of worker households and mixed group households (i.e. the group that is partly working and partly retired, and decides upon an optimal retirement age). A strength of the model is that it is able to capture life-cycle profiles of earnings and unemployment risks. The model contains search unemployment as pioneered by Mortensen (1986). We use a static search model as in Boone and Bovenberg (2002) in contrast to the dynamic model as Pissarides (2000), for example. The static model is simpler yet it captures the essential insights of the dynamic one. Firms post an optimal number of vacancies and unemployed persons choose an optimal search effort. Vacancies and individuals are allocated by job matching. Subsequently, the firm decides whether to keep the worker or fire her. Due to the structure of the model, we can explain age- and skill-dependent unemployment rates. Agents are endowed with labour productivity $\theta^a_{\alpha,t}$. This results from general productivity of a household, $\theta^{H,a}_{\alpha,t}$ due to past training and education and firm-specific skills $\theta^{F,a}_t$. Total labour productivity of an individual is given by $\theta^a_{\alpha,t} = \theta^{H,a}_{\alpha,t} \cdot \theta^{F,a}_{\alpha,t}$. Firms decide on the optimal amount of firm-sponsored training, which is determined by the increase of labour productivity and the costs associated with training. Employers and employees bargain over wages, given the bargaining power of workers and firms. In our Nash-bargaining setting, they take into account labour productivity and the outside option of the employee, which mainly depends on the unemployment replacement rate and the value of home production. In addition, institutional settings, e.g. the tax and benefit system are taken into account. At the beginning of each period, an individual of age group a and skill group i who participates in the labour market is assumed to have a job without searching with an exogenous probability ε^a (we suppress the skill index i again). With probability $(1-\varepsilon^a)$ she has to search for a job and with probability f^a per unit of search effort s^a_α there is a match with an open vacancy. Therefore, the hiring rate hir^a_α consists of those individuals who have a job without searching plus those who are matched with firms' vacancies. Subsequently, firms optimally choose the number of workers they want to fire. Only a share p^a_{man} of hired workers hir^a_α remain employed. As a result, the stock of unemployed consists of not matched job-seekers and matched job-seekers who are fired (with probability $(1-p^a_{man})$),

$$\begin{aligned} hir^a_\alpha &= \varepsilon^a + (1-\varepsilon^a)\, s^a_\alpha f^a, \qquad (25)\\ 1-u^a_\alpha &= p^a_{man} hir^a_\alpha, \\ u^a_\alpha &= (1-p^a_{man})\, hir^a_\alpha + (1-s^a_\alpha f^a)(1-\varepsilon^a) = \qquad (26)\\ &= \varepsilon^a (1-p^a_{man}) + (1-s^a_\alpha f^a p^a_{man})(1-\varepsilon^a). \qquad (27)\end{aligned}$$

f^a and p^a_{man} are determined endogenously in the model but from an individual perspective, they are taken as given. Posting vacancies and firing workers are age- and skill-dependent, therefore they differ across groups.

In any given period, workers pursue a sequence of activities: (i) Participation decision at the beginning of period; if households participate in the labour market, they incur a participation cost $\varphi^P(\delta^a_\alpha)\,\theta^a_\alpha$. (ii) Participants are immediately allocated to a job with probability ε^a, while they must search for a job with probability $1-\varepsilon^a$. In the latter case, they invest an effort s^a_α for search activities and incur search costs $\varphi^S(s^a_\alpha)\,\theta^a_\alpha$. Firms post vacancies v^a and incur job creation costs κ. (iii) Matching; job seekers and open vacancies are matched if job search is successful; (iv) Wage bargaining; (v) firing occurs with probability $1-p^a_{man}$. Thus, in case of participation, the probability of being employed (resp. being unemployed) is given by $1-u^a_\alpha$ (resp. u^a_α); (vi) production. If unemployed, agents engage in home production. If they have a job, they choose total hours 'worked', $L^a_\alpha = l^a_\alpha + e^a_\alpha$, incurring an effort cost $\varphi^L(L^a_\alpha)\,\theta^a_\alpha$.[9] They allocate their total time budget to productive work l^a_α and training e^a_α. As usual, we solve by backward induction.

In choosing consumption, savings and work related activities today, agents anticipate how these decisions affect their welfare during later life-cycle stages. The value function of the workers is given by:

$$V\left(A^a_{\alpha,t}, P^a_{\alpha,t}, \theta^{H,a}_{\alpha,t}\right) = \max_{C^a_{\alpha,t}, l^a_{\alpha,t}, s^a_{\alpha,t}, \delta^a_{\alpha,t}, e^a_{\alpha,t}} \left[(Q^a_{\alpha,t})^\rho + \gamma^a \beta \left(G\bar{V}^a_{\alpha,t+1}\right)^\rho\right]^{1/\rho}, \qquad (28)$$

where $Q_{\alpha,t}$ is the effort adjusted level of consumption. Individuals maximize their utility by choosing the optimal number of hours worked $l^a_{\alpha,t}$ if employed, the optimal search effort $s^a_{\alpha,t}$ if looking for a job, the optimal participation rate on the labour market $\delta^a_{\alpha,t}$, the optimal training effort $e^a_{\alpha,t}$, and the optimal consumption level $C^a_{\alpha,t}$. The effort adjusted level of consumption takes into account the disutility of hours worked, job search, and participation and the value of home production:

$$\begin{aligned} Q^a_\alpha &= C^a_\alpha - \theta^a_{\alpha,t}\bar{\varphi}^a_\alpha, \qquad (29a)\\ \bar{\varphi}^a_\alpha &= \left\{\delta^a_\alpha \bar{\delta}^a \left[(1-u^a_\alpha)\,\varphi^L(L^a_\alpha) - u^a_\alpha h^a_u + (1-\varepsilon^a)\,\varphi^S(s^a_\alpha)\right] + \bar{\delta}^a\left[\varphi^P(\delta^a_\alpha) - (1-\delta^a_\alpha)\, h^a_\delta\right]\right\}. \qquad (29b)\end{aligned}$$

In assuming preferences that are additively separable in consumption C and job related efforts, we eliminate income effects on labour supply. Again, $\bar{\delta}^a$ is the share of workers in age group a that is not

[9] Actually, all the effort costs are multiplied by individual productivity θ^H_α only and are not multiplied by firm-specific skills $\theta^{F,a}$ in the model. In order to simplify notation throughout the model documentation, we multiply these costs by labour productivity θ^a_α and 'normalize' effort costs by dividing them by $\theta^{F,a}$ (e.g. $\varphi^{L,true}\theta^{H,a}_\alpha = \left(\varphi^{L,true}/\theta^{F,a}\right)\theta^a_\alpha = \varphi^L\theta^a_\alpha$).

disabled. All efforts differ by age but are independent of history, i.e. $l^a_{\alpha,t} = l^a_t$, as will become evident from the solution. This symmetry is essential for aggregation. h^a_u and h^a_δ represent the value of home production of an individual that is unemployed resp. not participating, and increase the utility of non-employment. These values are multiplied by θ^a_α, so that home production increases with productivity of a person.[10]

There are three different states for each member of a household, i.e. non-participation, unemployed or employed. If one had to take into account all biographies and all these different states separately, the optimization problem would be impossible to solve. For this reason we make an assumption which is common in many applications, namely the income pooling assumption, based on Andolfatto (1996). He assumes that there exists the possibility for a household to insure against the uncertainty of being unemployed in a period. He shows that in case of a concave first argument in the value function and a fully competitive environment in the insurance market[11] it is optimal for the households to insure fully against the risk of unemployment. Accordingly, all members of a household pool their income and divide it among them equally.

The individual stock of assets accumulates according to:

$$G\gamma^a A^a_{\alpha,t+1} = R^\tau_{t+1}\left[A^a_{\alpha,t} + y^a_{\alpha,t} + z^a_t + iv^a_t - pc^a_t C^a_{\alpha,t}\right], \tag{30}$$

which is analogous to the retirees, see equation (10).

Labour related pooled income y^a_α and effort-adjusted pooled income $\bar{y}^a_\alpha$ are given by:

$$(W): \quad y^a_\alpha = \left\{\delta^a_\alpha\left[(1-u^a_\alpha)\, w^a_{\alpha,net} + u^a_\alpha b^a_\alpha + w^a l^a_\alpha hir^a_\alpha sev^a_\alpha\right] + (1-\delta^a_\alpha)\, z^a_{npar}\right\}\bar{\delta}^a\theta^a_\alpha + \tag{31a}$$
$$+\left(1-\bar{\delta}^a\right) DP^a,$$

$$(M): \quad y^a_\alpha = \delta^a_\alpha\bar{\delta}^a\left[(1-u^a_\alpha)\, w^a_{\alpha,net} + u^a_\alpha b^a_\alpha + w^a l^a_\alpha hir^a_\alpha sev^a_\alpha\right]\theta^a_\alpha + \bar{\delta}^a\,(1-\delta^a_\alpha)\, z^a_{npar} \tag{31b}$$
$$+\left(1-\bar{\delta}^a\right) DP^a,$$

$$(M): \quad z^a_{npar} = (1-\tau^{p,a})\left[\left(1+\sigma^P_t\left(\delta^a_\alpha\right)\right) P^a_\alpha \nu^a_1 + P_0\right] = P^a_{\alpha,net}, \quad \sigma^P = \left(\delta^a_\alpha - \delta^{R,P}\right)\sigma^P_1 + \sigma^P_0.$$

$$w^a_{\alpha,net} = (1-\tau^a)\, w^a l^a_\alpha + z^a_w,$$

$$sev^a_\alpha = \tau^{S,a}\,(1-p^a_{man})\, fac^a\left(1-t^{w,S,a}\right),$$

$$\bar{y}^a_\alpha = y^a_\alpha - pc^a\theta^a_{\alpha,t}\bar{\varphi}, \tag{31c}$$

where (W) stands for all workers groups and (M) for the mixed group. Income consists of net of tax wage, unemployment benefits, severance payments, and social assistance for not participating individuals. If employed, agents get net wage per efficiency unit $w^a_{\alpha,net}$, and the observed net wage of an agent with productivity θ^a_α is $\theta^a_\alpha w^a_{\alpha,net}$. If unemployed, they receive an unemployment benefit b^a_α (per efficiency unit). If fired, which occurs with probability $(1-p^a_{man})$, an individual receives severance payments from the firm, amounting to a share $\tau^{S,a}$ of gross wage income and which may be subject to income

[10] The model is flexible here as h^a_u and h^a_δ are age- and skill-dependent. Again, we normalize home production by firm-specific skills, so that it increases 'only' with individual productivity.

[11] This means there are no profits and no overhead costs in this market.

taxation.[12] τ^a is the average tax rate and includes the income tax rate $t^{w,a}$ as well as social security contributions $t^{sscW,a}$. As for the retirees, the factor $x^{W,a}$ determines the amount to which social security contributions are deductible for income taxation, hence $(1-\tau^a) = \left(1 - t^{w,a}\left(1 - x^{W,a} t^{sscW,a}\right) - t^{sscW,a}\right)$. [13] Furthermore, the parameter z_w^a may also include taxes or social security contributions that do not depend on income, but only depend on the labour market status.

Individuals of younger working groups who do not participate in the labour market receive a transfer z_{npar}^a per efficiency unit. The mixed group is special since it consists of workers and retirees. The composition depends on the endogenous retirement decision. Upon retirement, labour income is replaced by public pension benefits. For the calculation of these payments, see the following discussion on pension claims. In case of disability, which occurs with exogenous probability $\left(1-\bar{\delta}^a\right)$ and might occur for younger age groups as well, the household receives disability pension payments which are partly earnings-related and partly flat, $DP^a = \left(1-\tau^{pd}\right)\left(P_{\alpha,t}^a \nu_2^a + P_0^{DP}\right)$.

Individual productivity θ^H grows with new production $F\left(\delta_\alpha^a \bar{\delta}^a \left(1-u_{\alpha,t}^a\right) e_{\alpha,t}^a\right)$ for an employed person and depreciates with a factor $\delta^{H,a}$.[14]

$$G\theta_{\alpha,t+1}^{H,a} = \left(\left(1-\delta^{H,a}\right) + F_{\alpha,t}^a\right)\theta_{\alpha,t}^{H,a}. \tag{32}$$

When persons switch to the next age group, the stocks are $\theta_{\alpha',t+1}^{H,a+1} = \theta_{\alpha,t+1}^{H,a}$, $A_{\alpha',t+1}^{a+1} = A_{\alpha,t+1}^a$, and $P_{\alpha',t+1}^{a+1} = P_{\alpha,t+1}^a$.

Because of lack of data and empirical estimates, we assume that only employed individuals undergo training which improves their productivity. Otherwise, we would need data about the breakdown of life-cycle wage profiles according to labour productivity resulting from general training of employed people, general training of unemployed people and firm-sponsored training. It is often assumed in the literature that training for unemployed people increases matching efficiency.[15,16]

Unemployment Payments Unemployed individuals receive unemployment benefits b_α^a per efficiency unit,

$$b_\alpha^a = \xi_1\left(b\left(1-\tau^{u,a}\right)w^a l_\alpha^a + z_u^a\right) + \left(1-\xi_1\right)b^{0,a} \tag{33}$$

that may be partly indexed to labour income, which is captured by the parameter ξ_1. For full indexation, $\xi_1 = 1$.[17] b is the replacement rate for the unemployment payments. $\tau^{u,a}$ is the average tax rate for an

[12]We apply a factor fac^a to that amount, so that we can correct the payment if it is not dependent on gross labour income but rather on net labour income.

[13]Potentially, the income tax rate $t^{w,a}$ consists of the 'true' income tax rate $t^{w,inc,a}$ and a 'tax rate' $t^{w,soc,a}$ induced by social assistance systems, $t^{w,a} = t^{w,inc,a} + t^{w,soc,a}$. Individuals with low income receive social assistance z_w^a, but they may lose (some of) it if they increase their labour income. This effectively imposes a tax on higher income, captured by the social assistance 'tax rate' $t^{w,soc,a}$.

[14]We simply write $F_{\alpha,t}^a$ or F_t^a for $F\left(\delta_\alpha^a \bar{\delta}^a \left(1-u_{\alpha,t}^a\right) e_{\alpha,t}^a\right)$.

[15]The government may encourage private training by providing additional facilities, which might increase the effectiveness parameter of the production function of human capital formation F.

[16]Most models with human capital formation assume either learning-by-doing or on-the-job-training, and don't include training of unemployed or non-participating individuals or firm-specific skills.

[17]The literature provides evidence that the extent to which unemployment payments are indexed to wages has an important role in determining labour market outcomes. For example, it has an influence on wage setting in the Nash-bargaining

unemployed worker, that may differ from the average tax rate τ^a of an employed individual.

Pension System Individual pension claims evolve according to

$$(W): \quad GP^a_{\alpha,t+1} = R^{P,a}\left[\delta^a_\alpha \bar{\delta}^a \left[(1-u^a_\alpha+u^a_\alpha b_1)\, m^a w^a \theta^a_\alpha l^a_\alpha + m^a_1 \theta^a_\alpha\right] + \left(1-\bar{\delta}^a\right) X^a_0 + \sigma^P P^a_{\alpha,t}\right] \quad (34a)$$

$$(M): \quad GP^a_{\alpha,t+1} = R^{P,a}\left[\sigma^M \left(\delta^a_a\right) \bar{\delta}^a \left[(1-u^a_\alpha+u^a_\alpha b_1)\, m^a w^a \theta^a_\alpha l^a_\alpha + m^a_1 \theta^a_\alpha\right] + \left(1-\bar{\delta}^a\right) X^a_0\right] + \quad (34b)$$

$$+R^{P,a}\left(\sigma^P\left(\delta^a_a\right)+1\right) P^a_{\alpha,t},$$

$$\sigma^M = \left(\delta^\alpha_\alpha - \delta^{R,M}\right)\sigma^M_1 + \sigma^M_0,$$

$$(W): \quad \sigma^P = \left(1-\delta^{P,a}\right),$$

$$(M): \quad \sigma^P = \left(\delta^a_\alpha - \delta^{R,P}\right)\sigma^P_1 + \sigma^P_0,$$

$$X^a_0 = \left[1-u^a_0+u^a_0 b_1\right] m^a w^a_0 \theta^a_0 l^a_0 + m^a_1 \theta^a_0.$$

Pension entitlements $P^a_{\alpha,t+1}$ in period $t+1$ are equal to the stock in period t plus payments to the pension system in t. The system pays a notional interest R^P_{t+1} on past entitlements (which may be equal to G or R). The factor m^a and labour income determine the yearly increase of the determination base for pensions, i.e. individuals increase their future pension benefits by contributing today. Unemployed income can also raise pension entitlements with a factor b_1. However, the increase of the pension assessment base is determined by gross earnings $lw\theta$ prior to unemployment. m_1 captures the fact that the increase of pensions is partly dependent on participation on the labour market and not on income in some of the countries modelled. The term $\left(1-\bar{\delta}^a\right) X^a_0$ accounts for the fact that a low pension stock of an individual retiring because of disability can be topped up in order to reduce the poverty risk of disabled individuals.

For the mixed group, the terms σ^j capture institutional retirement incentives which compensate for continued contributions (σ^M) and foregone pension benefits (σ^P) when retirement is postponed. If $\sigma^j_1 > 0$, postponed retirement is rewarded by additional pension supplements à la Gruber and Wise (2005) while early retirement before the statutory retirement age $\delta^{R,P}$ is penalized by pension discounts. For the other age groups, the term $\sigma^P = \left(1-\delta^{P,a}\right)$ captures the feature of pension systems that earlier pension contributions might 'depreciate'.

3.2.1 Optimization

The individual maximizes the value function (28) by choosing the optimal intertemporal allocation of consumption, the participation rate resp. retirement age, search, working and training efforts, subject to the accumulation equations for assets (30), pension rights (34a) resp. (34b), and productivity (32).

For the envelope conditions, it is useful to define the shadow price of the productivity level as[18]

environment.

[18]The other shadow prices are defined similar to those of the retirees.

$$\chi^a_{\alpha,t} \equiv \frac{dV^a_{\alpha,t}}{d\theta^{H,a}_{\alpha,t}} \left(V^a_{\alpha,t}\right)^{\rho-1},$$

$$\bar{\chi}^a_{\alpha,t+1} \equiv \left[\omega^a \frac{dV^a_{\alpha,t+1}}{d\theta^{H,a}_{\alpha,t+1}} + (1-\omega^a)\frac{dV^{a+1}_{\alpha',t+1}}{d\theta^{H,a+1}_{\alpha',t+1}}\right]\left(\bar{V}^a_{\alpha,t+1}\right)^{\rho-1}.$$

The envelope conditions are given by:

$$A \;:\; \eta^a_{\alpha,t} = \beta R^\tau_{t+1}\bar{\eta}^a_{\alpha,t+1}G^{\rho-1}, \tag{35a}$$

$$(W):P \;:\; \tilde{\lambda}^a_{\alpha,t} = \frac{\lambda^a_{\alpha,t}}{\eta^a_{\alpha,t}} = \left(1-\bar{\delta}\right)\left(1-\tau^{pd}\right)\nu^a_2 + \frac{\gamma^a R^{P,a}\sigma^{P,a}}{R^\tau}\frac{\bar{\lambda}^a_{\alpha,t+1}}{\bar{\eta}^a_{\alpha,t+1}},$$

$$(M):P \;:\; \tilde{\lambda}^a_{\alpha,t} = \frac{\lambda^a_{\alpha,t}}{\eta^a_{\alpha,t}} = \bar{\delta}\,(1-\delta)\,(1-\tau^p)\,\nu_1\left(1+\sigma^{P,a}\right) + \tag{35b}$$

$$+\left(1-\bar{\delta}\right)\left(1-\tau^{pd}\right)\nu^a_2 + \frac{\gamma^a\left(R^{P,a}\left(\sigma^{P,a}+1\right)\right)}{R^\tau}\frac{\bar{\lambda}^a_{\alpha,t+1}}{\bar{\eta}^a_{\alpha,t+1}},$$

$$(W):\theta^H \;:\; \tilde{\chi}^a_{\alpha,t} = \frac{\chi^a_{\alpha,t}}{\eta^a_{a,t}} = \left(-pc^a\bar{\varphi}^a_\alpha + \tilde{y}^{W,a}_\alpha + \delta^a_\alpha\bar{s}^a_\alpha\bar{\delta}^a\right)\theta^{F,a} + \tag{35c}$$

$$+\left[\left(1-\delta^{H,a}\right)+F^a_\alpha\right]\frac{\gamma^a\bar{\chi}^a_{\alpha,t+1}}{R^\tau\bar{\eta}^a_{\alpha,t+1}},$$

$$(M):\theta^H \;:\; \tilde{\chi}^a_{\alpha,t} = \frac{\chi^a_{\alpha,t}}{\eta^a_{a,t}} = \left(-pc^a\bar{\varphi}^a_\alpha + \tilde{y}^{M,a}_\alpha + \sigma^{M,a}_\alpha\bar{s}^a_\alpha\bar{\delta}^a\right)\theta^{F,a} + \tag{35d}$$

$$+\left[\left(1-\delta^{H,a}\right)+F^a_\alpha\right]\frac{\gamma^a\bar{\chi}^a_{\alpha,t+1}}{R^\tau\bar{\eta}^a_{\alpha,t+1}},$$

$$\tilde{y}^{W,a}_\alpha = \delta^a_\alpha\bar{\delta}^a\left[(1-u^a_\alpha)\,w^a_{net,\alpha} + u^a_\alpha b^a_\alpha + w^a l^a_\alpha hir^a_\alpha sev^a_\alpha\right] + (1-\delta^a_\alpha)\,\bar{\delta}^a z^a_{npar},$$

$$\tilde{y}^{M,a}_\alpha = \delta^a_\alpha\bar{\delta}^a\left[(1-u^a_\alpha)\,w^a_{net,\alpha} + u^a_\alpha b^a_\alpha + w^a l^a_\alpha hir^a_\alpha sev^a_\alpha\right],$$

$$\bar{s}^a = (1-u^a_\alpha + u^a_\alpha b_1)\,m^a w^a l^a_\alpha\frac{\gamma^a R^{P,a}}{R^\tau}\frac{\bar{\lambda}^a_{\alpha,t+1}}{\bar{\eta}^a_{\alpha,t+1}}.$$

The first order conditions are given by:

$$C \;:\; \left(Q^a_{\alpha,t}\right)^{\rho-1} = \beta R^\tau_{t+1}\bar{\eta}^a_{\alpha,t+1}G^{\rho-1}pc^a_t, \tag{36a}$$

$$l \;:\; pc^a\varphi^{L\prime} = (1-\hat{\tau}^a)\,w^a = \left((1-\tau^a) + gain^a_u + gain^a_p + gain^a_{sev}\right)w^a, \tag{36b}$$

$$(W):s \;:\; pc^a\varphi^{S\prime}_\alpha = f^a\left\{\begin{array}{c} p^a_{man}\left[w^a l^a_\alpha\Gamma^{a,W} - pc^a\left(\varphi^L_\alpha + h^a_u\right) + z^a_w - \xi_1 z^a_u - (1-\xi_1)\,b^{0,a}\right] + \\ p^a_{man}\frac{\gamma^a e^a_\alpha F^a_{E,\alpha}}{R^\tau}\frac{\bar{\chi}^a_{\alpha,t+1}}{\bar{\eta}^a_{\alpha,t+1}} + w^a l^a_\alpha\tau^{S,a}fac^a\left(1-t^{w,S,a}\right)\end{array}\right\}, \tag{36c}$$

$$(M):s \;:\; pc^a\varphi^{S\prime}_\alpha = f^a\left\{\begin{array}{c} p^a_{man}\left[w^a l^a_\alpha\Gamma^{a,M} - pc^a\left(\varphi^L_\alpha + h^a_u\right) + z^a_w - \xi_1 z^a_u - (1-\xi_1)\,b^{0,a}\right] + \\ p^a_{man}\frac{\gamma^a e^a_\alpha F^a_{E,\alpha}}{R^\tau}\frac{\bar{\chi}^a_{\alpha,t+1}}{\bar{\eta}^a_{\alpha,t+1}} + w^a l^a_\alpha\tau^{S,a}fac^a\left(1-t^{w,S,a}\right)\end{array}\right\} \tag{36d}$$

$$
\begin{aligned}
(W):\delta \quad &: \quad pc^a\varphi_\alpha^{P\prime} = (1-u_\alpha^a)\left(w_{net,\alpha}^a - pc^a\varphi_\alpha^L\right) + u_\alpha^a b_\alpha^a + w^a l_\alpha^a hir_\alpha^a sev_\alpha^a - \qquad (36e)\\
&\quad -(1-\varepsilon)\varphi_\alpha^S pc^a - z_{npar}^a + (1-u_\alpha^a)\frac{\gamma^a e_\alpha^a F_{E,\alpha}^a}{R^\tau}\frac{\bar{\chi}_{\alpha,t+1}^a}{\bar{\eta}_{\alpha,t+1}^a} + \bar{s}_\alpha^a + pc^a\left(u_\alpha^a h_u^a - h_\delta^a\right),\\
(M):\delta \quad &: \quad \theta_\alpha^a pc^a\varphi_\alpha^{P\prime} = \theta_\alpha^a\left[(1-u_\alpha^a)\left(w_{net,\alpha}^a - pc^a\varphi_\alpha^L\right) + u_\alpha^a b_\alpha^a + w^a l_\alpha^a hir_\alpha^a sev_\alpha^a\right] + \qquad (36f)\\
&\quad +\theta_\alpha^a\left[(1-u_\alpha^a)\frac{\gamma^a e_\alpha^a F_{E,\alpha}^a}{R^\tau}\frac{\bar{\chi}_{\alpha,t+1}^a}{\bar{\eta}_{\alpha,t+1}^a} + \sigma_1^M\bar{s}_\alpha^a - (1-\varepsilon)\varphi_\alpha^S pc^a + pc^a\left(u_\alpha^a h_u^a - h_\delta^a\right)\right] +\\
&\quad +\frac{\gamma^a R^{P,a}}{\bar{\delta}^a R^\tau}\frac{\bar{\lambda}_{\alpha,t+1}^a}{\bar{\eta}_{\alpha,t+1}^a}\sigma_1^{P,a}P_\alpha^a - P_{\alpha,net} + (1-\delta^a)\sigma_1^{P,a}P_\alpha^a\nu_1(1-\tau^p),\\
e \quad &: \quad pc^a\varphi_\alpha^{L\prime} = \frac{\gamma^a F_{E,\alpha}}{R^\tau}\frac{\bar{\chi}_{\alpha,t+1}^a}{\bar{\eta}_{\alpha,t+1}^a}, \qquad (36g)\\
gain_u^a \quad &= \quad (1-\tau^{u,a})\frac{u_\alpha^a}{1-u_\alpha^a}\xi_1 b,\\
gain_{sev}^a \quad &= \quad \frac{hir_\alpha^a sev_\alpha^a}{(1-u_\alpha^a)},\\
(W) \quad &: \quad gain_p^a = \frac{R^{P,a}\gamma^a m}{R^\tau}\left[1+\frac{u_\alpha^a}{1-u_\alpha^a}b_1\right]\frac{\bar{\lambda}_{\alpha,t+1}^a}{\bar{\eta}_{\alpha,t+1}^a}\\
(M) \quad &: \quad gain_p^a = \frac{R^{P,a}\gamma^a m}{R^\tau}\left[1+\frac{u_\alpha^a}{1-u_\alpha^a}b_1\right]\frac{\bar{\lambda}_{\alpha,t+1}^a}{\bar{\eta}_{\alpha,t+1}^a}\frac{\sigma^M}{\delta_\alpha^a},\\
\Gamma^{a,W} \quad &= \quad \left[(1-\tau) - (1-\tau^u)\xi_1 b + (1-b_1)m\frac{\gamma^a R^{P,a}}{R^\tau}\frac{\bar{\lambda}_{\alpha,t+1}^a}{\bar{\eta}_{\alpha,t+1}^a} - \tau^{S,a}fac^a\left(1-t^{w,S,a}\right)\right],\\
\Gamma^{a,M} \quad &= \quad \left[(1-\tau) - (1-\tau^u)\xi_1 b + (1-b_1)m\frac{\gamma^a R^{P,a}}{R^\tau}\frac{\bar{\lambda}_{\alpha,t+1}^a}{\bar{\eta}_{\alpha,t+1}^a}\frac{\sigma^M}{\delta_\alpha^a} - \tau^{S,a}fac^a\left(1-t^{w,S,a}\right)\right],\\
\bar{s}_\alpha^a \quad &= \quad \left((1-u_\alpha^a + u_\alpha^a b_1)m^a w^a l_\alpha^a + m_1^a\right)\frac{\gamma^a R^{P,a}}{R^\tau}\frac{\bar{\lambda}_{\alpha,t+1}^a}{\bar{\eta}_{\alpha,t+1}^a}.
\end{aligned}
$$

Remark 6 *It can be shown that l_α, s_α, (and thus u_α), e_α and participation δ_α of workers do not depend on the biography α,hence $l = l_\alpha$, etc. The same holds for the shadow price variables λ_α, η_α and χ_α. This is crucial for the aggregation.*

Remark 7 *However, the heterogeneity in previously accumulated pension entitlements and productivity levels, P_α and θ_α creates heterogeneity in the retirement decision (see 36f). This heterogeneity must be avoided to allow for analytical aggregation. We thus assume a two stage decision structure. In a first stage, people collectively decide on a uniform retirement age. This is, of course, an approximation which successfully captures retirement incentives and yet retains symmetry. In stage two, people decide on job search and work during their active part of the mixed phase. We solve backwards and first derive work related choices, given a retirement age.*

The first order condition for consumption (36a) is the same as for retirees, except that it determines effort-adjusted consumption Q^a instead of 'pure' consumption C^a. The first order conditions for 'labour supply', i. e. the number of hours worked l (36b), search intensity s (36c resp. 36d), participation resp. retirement δ (36e resp. 36f), and training e (36g) equate the marginal disutility with the marginal gains from providing an additional unit of time to these different time use possibilities. Given the functional forms (see Section 10), marginal disutilities increase with additional time devoted to one of these possibilities if supply elasticities are positive. Thus, for all these margins, an increase of the marginal gains on the right hand side of these equations would imply an increase of the corresponding time input.

The first order condition with respect to effort (36b) equates the additional effective net income of an additional hour of work with the utility costs. The effective tax rate $\hat{\tau}^a$ is lower than the statutory rate since part of increased work effort also raises unemployment compensation, future pension benefits and potential severance payments. The extra benefits for the unemployment system $gain_u^a$, the pension system $gain_p^a$ and severance payments $gain_{sev}^a$ are expressed as a share of wages and are subtracted from the statutory tax rate.[19] In that sense and ceteris paribus, higher future pension benefits, unemployment and severance payments have a positive impact on the number of hours worked.

The first order condition with respect to the search intensity (36c) resp. (36d) equates the marginal disutility of searching with the expected additional effective labour income less the outside option of the household member when spending an additional search unit. The increase of search intensity improves the chance to find a job one by one.[20] In Γ^a all terms that are proportional to gross wage income $w^a l^a$ are collected. The term Γ^a captures the gains and losses when an individual moves from unemployment into employment by accepting a job that pays a gross wage $w^a l^a$ per effective productivity unit. The first term of Γ^a gives the increase in net income after taxes when accepting a job offer but this gain is reduced by the factor $(1-\tau^u)\,\xi_1 b$ due to the foregone unemployment benefit (the share which is proportional to wage income).[21] The second term shows the gain in terms of higher pension rights resulting from being employed. Again, this gain is reduced by a factor $1-b_1$ due to the pension rights that stem from unemployment periods. The difference between labour and unemployment income is further decreased by the effort costs of working $pc^a\varphi^L$, the value of home production $pc^a h_u^a$ if unemployed and by the part of unemployment benefits that is not indexed to wages $(1-\xi_1)\,b^{0,a}$. Differences in 'lump-sum transfers' when being employed and being unemployed (receiving wage-indexed unemployment benefits) are reflected in the term $z_w^a - \xi_1 z_u^a$. Furthermore, the condition for optimal search effort reflects the future income gains from finding a job today that results from future productivity gains. There is no training and therefore a loss of productivity during periods of unemployment. Finally, with probability $(1-p_{man}^a)$, an individual is fired by the firm and receives severance payments, making it more attractive to look for a job in order to qualify for these severance payments. For the wage-bargaining problem, it will be useful to rewrite (36c) resp. (36d) to

$$\begin{aligned} pc^a \varphi_\alpha^{S\prime} &= f^a \left\{ \left[p_{man}^a l_\alpha^a \Gamma^a + l_\alpha^a \tau^{S,a} fac^a \left(1 - t^{w,S,a}\right)\right] \left[w^a - w_-^a \right] \right\}; \\ w_-^a &= p_{man}^a \frac{\left(pc^a \left(\varphi_\alpha^L + h_u^a\right) - z_w^a + \xi_1 z_u^a + (1-\xi_1)\, b^{0,a} - \frac{\gamma^a e_\alpha^a F_{E,\alpha}^a}{R^\tau} \frac{\bar{\chi}_{\alpha,t+1}^a}{\bar{\eta}_{\alpha,t+1}^a} \right)}{\left[p_{man}^a l_\alpha^a \Gamma^a + l_\alpha^a \tau^{S,a} fac^a \left(1 - t^{w,S,a}\right)\right]}, \end{aligned} \tag{37}$$

[19] However, to derive the total effective tax rate on labour income, contribution rates of employers must be taken into account.

[20] From an individual perspective, the probability f of finding a job per unit of search intensity is exogenous. The probability is determined in equilibrium by aggregate search intensity and aggregate vacancies.

[21] Note here that institutional design may provide different incentives on different margins. For example, ceteris paribus, higher replacement rates can provide positive incentives on the intensive margin (number of hours worked) as this increases $gain_u$. However, this lowers search incentives given the increase of the outside option (which is captured in the term Γ^a).

so that the search intensity of an unemployed depends on the wage w^a and her reservation wage w^a_-.

The first order condition for training (36g) equates marginal effort costs of training with the additional future benefits of higher productivity.

Endogenous Participation and Retirement For the working groups, the interpretation of the first order condition with respect to participation is fairly easy. The marginal utility cost of participation has to be equal to the marginal expected gain of participation. Higher participation provides labour income if employed, unemployment income if unemployed and severance payments if fired. Furthermore, effort costs (number of hours worked, search effort, training), home production and effects of the pension system and of additional productivity in the future are taken into account.

The interpretation of the first order condition of retirement (36f) is similar. By retiring later, individuals receive labour and unemployment income, however they give up pension benefits. In addition, the system may give rise to positive participation incentives in the sense of Gruber and Wise (2005) if later retirement is rewarded (resp. early retirement penalized), which are also taken into account by the individuals (see σ). In our formulation, the household members of a skill group jointly decide upon the optimal retirement age. An *individual* optimal retirement condition cannot be implemented since the heterogeneity in $P^a_{\alpha,t}$ and $\theta^a_{\alpha,t}$ prevents a symmetric retirement age (i.e. an identical retirement age for all individuals within a particular group). Symmetric retirement is necessary for aggregation. It is therefore assumed that retirement is chosen on average (coordinated in the mixed group). This is, of course, an approximation which successfully captures retirement incentives and yet retains symmetry. The problem is set up new as

$$\max_\delta \sum\nolimits_\alpha \left[\left(Q^a_{\alpha,t}\right)^\rho + \gamma^a\beta\left(G\bar{V}^a_{\alpha,t+1}\right)^\rho\right]^{1/\rho} \cdot N^a_{\alpha,t}. \tag{38}$$

Similar to the aggregation procedure in section 5, we have $\sum_\alpha N^a_\alpha = N^a$,$\sum_\alpha \theta^a_\alpha N^a_\alpha = \Theta^a$,$\bar{P}^a = P^a/N^a$, $\bar{P}^a_{net} = P^a_{net}/N^a$ and $\bar{\theta}^a = \Theta^a/N^a$. The jointly chosen optimal retirement age is given by

$$\begin{aligned}(M):\delta \quad : \quad & \bar{\theta}^a pc^a\varphi^{P\prime} = \bar{\theta}^a\left[(1-u^a)\left(w^a_{net} - pc^a\varphi^L\right) + u^a b^a + w^a l^a hir^a sev^a\right] + \\ & +\bar{\theta}^a\left[(1-u^a)\frac{\gamma^a e^a F^a_E}{R^\tau}\frac{\bar{\chi}^a_{t+1}}{\bar{\eta}^a_{t+1}} + \sigma^M_1\bar{s}^a\right] + \frac{\gamma^a R^{P,a}}{\bar{\delta}^a R^\tau}\frac{\bar{\lambda}^a_{t+1}}{\bar{\eta}^a_{t+1}}\sigma^{P,a}_1\bar{P}^a - \\ & -\bar{P}^a_{net} + (1-\delta^a)\sigma^{P,a}_1\bar{P}^a\nu_1(1-\tau^p) + pc^a\bar{\theta}^a\left(u^a_\alpha h^a_u - h^a_\delta - (1-\varepsilon)\varphi^S\right).\end{aligned} \tag{39}$$

The interpretation is straightforward and corresponds exactly to equation (36f) above, except that it contains average stock variables instead of individual stock variables $P^a_{\alpha,t}$ and $\theta^a_{\alpha,t}$.

Box: Implicit participation/retirement tax:

To define the implicit tax rate $\hat{t}^{R,a}$ on participation/retirement (retirement tax), we first take the simplest case of fixed labour supply l^a without job search, implying zero effort costs $\bar{\varphi}^a$, eliminating any unemployment rate, and neglecting the possibility of disability pension ($\bar{\delta} = 1$). Suppose further that all dynamic linkages due to the accumulation of earnings related pension points are cut out, implying $P^a_\alpha = 0$ and leaving only a flat pension P^0 per capita. Let all taxes be zero except for a pension contribution rate t^p. One is left with an income $(1-t^p)\,w^a l^a \bar{\theta}^a$ if employed and total income $\delta^a(1-t^p)\,w^a l^a \bar{\theta}^a + (1-\delta^a)\,P^0$. In this simplest case, we can write the retirement decision as $\varphi^{P\prime}(\delta) = (1-t^p)\,w^a l^a \bar{\theta}^a - P^0 = \left(1-\hat{t}^{R,a}\right) w^a l^a \bar{\theta}^a$ where $\hat{t}^{R,a} \equiv t^p + P^0/\left(w^a l^a \bar{\theta}^a\right)$ is the implicit retirement tax. It is the sum of contribution rate t^p and pension replacement rate $P^0/\left(w^a l^a \bar{\theta}^a\right)$ and can, thus, be very high. This retirement tax corresponds to a labour market participation tax in the sense of Immervoll et al. (2007).

In our detailed model, the implicit retirement tax is more complicated. We define the participation tax relative to $(1-u^a)\,l^a w^a$. Per definition, if it is zero, the market outcome without public intervention is reproduced. Rewriting the first order conditions for participation and retirement, (36e) and (39), yields

$$
\begin{aligned}
(W) \quad : \quad & pc^a \varphi^{P\prime}(\delta) = (1-u^a)\left(l^a w^a - pc^a \varphi^L\right) - (1-\varepsilon)\,\varphi^S pc^a \\
& - \left[\left[t^{w,a} + t^{sscW,a} - x^{w,a} t^{w,a} t^{sscW,a}\right] l^a w^a (1-u^a) - b^a u^a\right] \\
& - z^a_{npar} + \bar{s}^a \\
& + (1-u^a)\left(\frac{\gamma^a e^a F^a_E}{R^\tau} \frac{\bar{\chi}^a_{t+1}}{\bar{\eta}^a_{t+1}} + z^a_w\right) + pc^a\left(u^a h^a_u - h^a_\delta\right) + w^a l^a hir^a sev^a, \\
(M) \quad : \quad & \bar{\theta}^a pc^a \varphi^{P\prime}(\delta) = \bar{\theta}^a\left[(1-u^a)\left(l^a w^a - pc^a \varphi^L\right) - (1-\varepsilon)\,\varphi^S pc^a\right] \\
& - \left[\left[t^{w,a} + t^{sscW,a} - x^{w,a} t^{w,a} t^{sscW,a}\right] l^a w^a (1-u^a) - b^a u^a\right] \bar{\theta}^a \\
& - \bar{P}^a_{net} + (1-\delta^a)\,\sigma^{P,a}_1 \bar{P}^a \nu_1 (1-\tau^p) + \bar{\theta}^a \sigma^M_1 \bar{s}^a + \frac{\gamma^a R^{P,a}}{\bar{\delta}^a R^\tau} \frac{\bar{\lambda}^a_{t+1}}{\bar{\eta}^a_{t+1}} \sigma^{P,a}_1 \bar{P}^a \\
& + \bar{\theta}^a (1-u^a)\left(\frac{\gamma^a e^a F^a_E}{R^\tau} \frac{\bar{\chi}^a_{t+1}}{\bar{\eta}^a_{t+1}} + z^a_w\right) + pc^a\left(u^a h^a_u - h^a_\delta\right) + w^a l^a hir^a sev^a.
\end{aligned}
$$

The interpretation of both equations is rather easy. The first line lists the net gains of working longer in the absence of government. The second line reflects the extra net taxes paid if retirement is postponed, consisting of wage tax and contribution rate minus foregone unemployment benefits and training costs. The third line lists foregone social resp. pensions benefits for the mixed group in the first term while the other terms reflect gains in pension benefits, which partly reflect pension supplements when retirement is postponed (Gruber-Wise incentives σ) in the mixed group. The fourth line lists the future gains of a productivity increase due to training if employed, the social assistance of an employed individual (social assistance of an unemployed individual is included in the unemployment benefit b^a), the change in the value of home production and severance payments received due to participating in the labour market. Expressing all terms relative to expected labour income $(1-u^a)\,l^a w^a$ for the workers and $(1-u^a)\,l^a w^a \bar{\theta}^a$ for the mixed group yields the implicit retirement tax,

$$
\begin{aligned}
(W) \quad &: \quad pc^a\varphi^{P\prime}(\delta) = \left(1-\hat{t}^{R,a}\right)(1-u^a)\,l^a w^a - (1-u^a)\,pc^a\varphi^L - (1-\varepsilon)\,\varphi^S pc^a, \\
(W) \quad &: \quad \hat{t}^{R,a} = \left[t^{w,a} + t^{sscW,a} - x^{w,a}t^{w,a}t^{sscW,a}\right] - \frac{b^a u^a}{(1-u^a)\,l^a w^a} + \\
&\quad + \frac{z^a_{npar}}{(1-u^a)\,l^a w^a} - \frac{\bar{s}^a}{(1-u^a)\,l^a w^a} \\
&\quad - \frac{1}{l^a w^a}\left(\frac{\gamma^a e^a F^a_E}{R^\tau}\frac{\bar{\chi}^a_{t+1}}{\bar{\eta}^a_{t+1}} + z^a_w\right) - \frac{pc^a\left(u^a h^a_u - h^a_\delta\right)}{(1-u^a)\,l^a w^a} - \frac{hir^a sev^a}{1-u^a}, \\
(M) \quad &: \quad pc^a\varphi^{P\prime}(\delta) = \left(1-\hat{t}^{R,a}\right)(1-u^a)\,l^a w^a - (1-u^a)\,pc^a\varphi^L - (1-\varepsilon)\,\varphi^S pc^a, \\
(M) \quad &: \quad \hat{t}^{R,a} = \left[t^{w,a} + t^{sscW,a} - x^{w,a}t^{w,a}t^{sscW,a}\right] - \frac{b^a u^a}{(1-u^a)\,l^a w^a} \\
&\quad + \frac{\bar{P}^a_{net} - (1-\delta^a)\,\sigma_1^{P,a}\bar{P}^a\nu_1(1-\tau^p) - \sigma_1^M\bar{s}^a\bar{\theta}^a - \frac{\gamma^a R^{P,a}}{\delta^a R^\tau}\frac{\bar{\lambda}^a_{t+1}}{\bar{\eta}^a_{t+1}}\sigma_1^{P,a}\bar{P}^a}{(1-u^a)\,l^a w^a\bar{\theta}^a} \\
&\quad - \frac{1}{l^a w^a}\left(\frac{\gamma^a e^a F^a_E}{R^\tau}\frac{\bar{\chi}^a_{t+1}}{\bar{\eta}^a_{t+1}} + z^a_w\right) - \frac{pc^a\left(u^a h^a_u - h^a_\delta\right)}{(1-u^a)\,l^a w^a} - \frac{hir^a sev^a}{1-u^a}.
\end{aligned}
$$

The implicit retirement (resp. participation) tax $\hat{t}^{R,a}$ summarizes all the disincentives for postponing retirement (resp. staying out of labour force) that are inherent in the system. It consists of the tax rates minus the sum of unemployment benefits, the benefit of training, and the social assistance of active individuals, plus the net loss in retirement (resp. non-participation) income. This net loss consists of the instantaneous foregone net of tax pension benefits (resp. social benefits). The other terms indicate that this loss is reduced by the increase in future pension benefits (partly on account of the Gruber/Wise mechanism for increased actuarial fairness). The present value of these future gains is captured by the shadow price $\bar{\lambda}/\bar{\eta}$ (which is also part of the term $\bar{s}^a$).

The modified Euler equation for the workers and the mixed group, that determines the intertemporal allocation of consumption is analogous to the retiree groups. However, instead of the allocation of consumption C^a_t, the allocation of effort-adjusted consumption Q^a_t is determined.

Proposition 6 ***(Euler equation for workers and the mixed group)*** *The Euler equation of workers is given by*

$$
Q^a_{\alpha,t}\left(pc^a_t\beta R^\tau_{t+1}\Omega^a_{t+1}\right)^\sigma = G\left[\begin{array}{c}\omega^a\left(pc^a_{t+1}\right)^\sigma Q^a_{\alpha,t+1} + \\ +(1-\omega^a)\left(pc^{a+1}_{t+1}\right)^\sigma Q^{a+1}_{\alpha',t+1}\Lambda^a_{\alpha,t+1}\end{array}\right]. \tag{40}
$$

Proof. analogous to retirees. ■

Total wealth of an individual consists of financial assets A^a_α, human wealth H^a_α, pension wealth S^a_α, and transfer wealth T^a_α. We define earnings related pension wealth S^a_α as the value of the *already accumulated* pension claims: $S^a_\alpha = \tilde{\lambda}^a P^a_\alpha$. An individual starts his life with a pension wealth of zero ($P^a_\alpha = S^a_\alpha = 0$ at the beginning of a lifetime). This makes clear that the pension wealth *is not* the same as the present value of all future pension benefits. However, it still holds that human wealth H^a_α plus pension wealth S^a_α is the sum of the present value of effort-adjusted income and the present value of earnings related pension benefits. Therefore, human wealth H_α^a in our model represents the present value of effort-adjusted income plus the difference of the present value of pension benefits and the value of already accumulated pension points (this difference is always greater or equal to zero). Thus, human wealth H_α^a, earnings related pension wealth S_α^a and transfer wealth T_α^a are given by:

$$
\begin{aligned}
(W): H_{\alpha,t}^a &= \left(\tilde{y}^W - pc^a\bar{\varphi} + \delta^a\bar{\delta}^a\bar{s}^a\right)\theta_\alpha^a + \frac{\gamma^a G}{R_{t+1}^\tau \Omega_{t+1}^a}\bar{H}_{\alpha,t+1}^a, && (41a)\\
(M): H_{\alpha,t}^a &= \left(\tilde{y}^M - pc^a\bar{\varphi} + \sigma^M\bar{\delta}^a\bar{s}^a\right)\theta_\alpha^a + \frac{\gamma^a G}{R_{t+1}^\tau \Omega_{t+1}^a}\bar{H}_{\alpha,t+1}^a, && (41b)\\
(W): S_{\alpha,t}^a &= \left(1-\bar{\delta}\right)\left(1-\tau^{pd}\right)\nu_2^a P_{\alpha,t}^a - \delta^a\bar{\delta}^a\bar{s}^a\theta_\alpha^a - && (41c)\\
&\quad -\frac{\gamma^a R^{P,a}}{R^\tau}\frac{\bar{\lambda}_{t+1}^a}{\bar{\eta}_{t+1}^a}\left(1-\bar{\delta}^a\right)X_0^a + \frac{\gamma^a G}{R_{t+1}^\tau \Omega_{t+1}^a}\bar{S}_{\alpha,t+1}^a, \\
(M): S_{\alpha,t}^a &= \left(1-\bar{\delta}\right)\left(1-\tau^{pd}\right)\nu_2^a P_\alpha^a + \bar{\delta}\left(1-\delta^a\right)\left(1-\tau^p\right)\nu_1\left(1+\sigma^P\right)P_\alpha - && (41d)\\
&\quad -\sigma^M\bar{\delta}^a\bar{s}^a\theta_\alpha^a - \frac{\gamma^a R^{P,a}}{R^\tau}\frac{\bar{\lambda}_{t+1}^a}{\bar{\eta}_{t+1}^a}\left(1-\bar{\delta}^a\right)X_0^a + \frac{\gamma^a G}{R_{t+1}^\tau \Omega_{t+1}^a}\bar{S}_{\alpha,t+1}^a, \\
(W): T_{\alpha,t}^a &= z_t^a + iv_t^a + \left(1-\bar{\delta}\right)\left(1-\tau^{DP}\right)P_0^{DP} + && (41e)\\
&\quad +\frac{\gamma^a R^{P,a}}{R^\tau}\frac{\bar{\lambda}_{t+1}^a}{\bar{\eta}_{t+1}^a}\left(1-\bar{\delta}^a\right)X_0^a + \frac{G\gamma^a}{R_{t+1}^\tau \Omega_{t+1}^a}\bar{T}_{\alpha,t+1}^a, \\
(M): T_{\alpha,t}^a &= z_t^a + iv_t^a + \left(1-\bar{\delta}\right)\left(1-\tau^{DP}\right)P_0^{DP} + \left(1-\delta\right)\left(1-\tau^P\right)P_0 + && (41f)\\
&\quad +\frac{\gamma^a R^{P,a}}{R^\tau}\frac{\bar{\lambda}_{t+1}^a}{\bar{\eta}_{t+1}^a}\left(1-\bar{\delta}^a\right)X_0^a + \frac{G\gamma^a}{R_{t+1}^\tau \Omega_{t+1}^a}\bar{T}_{\alpha,t+1}^a, \\
\tilde{y}^{W,a} &= \delta^a\bar{\delta}^a\left[\left(1-u^a\right)w_{net}^a + u^a b^a + w^a l_\alpha^a hir_\alpha^a sev_\alpha^a\right] + \left(1-\delta^a\right)\bar{\delta}^a z_{npar}^a, \\
\tilde{y}^{M,a} &= \delta^a\bar{\delta}^a\left[\left(1-u^a\right)w_{net}^a + u^a b^a + w^a l_\alpha^a hir_\alpha^a sev_\alpha^a\right], \\
\bar{s}^a &= \left(\left(1-u_\alpha^a + u_\alpha^a b_1\right)m^a w^a l_\alpha^a + m_1^a\right)\frac{\gamma^a R^{P,a}}{R^\tau}\frac{\bar{\lambda}_{t+1}^a}{\bar{\eta}_{t+1}^a}.
\end{aligned}
$$

Earnings related pension wealth S_α^a also includes disability pensions that are related to previous earnings. Furthermore, for the mixed group, it also includes benefits $\bar{\delta}\left(1-\delta^a\right)\left(1-\tau^p\right)\nu_1\left(1+\sigma^P\right)P_\alpha$ for retired households. Apart from lump sum transfers from the government and inter-vivo transfers, transfer wealth T_α^a includes those parts of pension benefits that are not linked to previous earnings.

Proposition 7 *Human capital and earnings related pension wealth are given by:*

$$
\begin{aligned}
H_\alpha^a &= \tilde{\chi}^a\theta_\alpha^{H,a} \text{ and} && (42)\\
S_\alpha^a &= \tilde{\lambda}^a P_\alpha^a, && (43)
\end{aligned}
$$

where $\tilde{\lambda}^a = \lambda^a/\eta^a$ *and* $\tilde{\chi}^a = \chi^a/\eta^a$.

Thus, human wealth and earnings related pension wealth are determined by the stock of pension rights and productivity and their respective shadow variables.

Proposition 8 ***(Policy of Workers and Mixed Group)*** *Consumption $Q^a_{\alpha,t}$ and indirect utility $V^a_{\alpha,t}$ are*

$$pc^a Q^a_\alpha = \frac{1}{\Delta^a}\left(A^a_\alpha + S^a_\alpha + H^a_\alpha + T^a_\alpha\right), \tag{44}$$

$$V^a_\alpha = \left(\Delta^a\right)^{\frac{1}{\rho}} Q^a_\alpha, \tag{45}$$

$$\Delta^a_t = 1 + \left(\frac{pc^a_t}{pc^a_{t+1}}\right)^{\sigma-1} \beta^\sigma \left(R^\tau_{t+1}\Omega^a_{t+1}\right)^{\sigma-1} \gamma^a \Delta^a_{t+1}.$$

Proof. analogous to retirees. ■

Effort adjusted consumption is determined by the marginal propensity to consume $1/\Delta^a$ and total wealth. Notice that the marginal propensity to consume is age-dependent. The longer remaining life expectancy of younger workers implies that they consume a lower share of their total wealth than older workers. Present value of welfare V^a_α is related to the marginal propensity to consume and effort-adjusted consumption.

3.3 Discrete Skill Choice

Up to now, the skill distribution was treated as given. However, there is also an endogenous skill choice in PuMA. We distinguish three skill classes, i.e. high-, medium- and low-skilled. As stated before, the optimization problem in the preceding subsections is analogous for all skills. Young individuals, who belong to the group of medium- and high-skilled persons are an exception. They are still in education and therefore inactive on the labour market. These individuals choose the consumption level optimally, but make no labour supply decisions. Indirect utility of a new agent born in t with skill i is $V^{i,1}_{t,t}$. This agent initially belongs to age group 1 with history $\alpha_1 = t$. Let $c^i > 0$ be the *incremental* educational effort costs that an agent with skill $i-1$ needs to invest to obtain skill i.[22] The discrete skill choice at the beginning of the life-cycle is[23]

$$i = \arg\max_{\tilde{i}} V^{\tilde{i},1}_{t,t} - \sum_{j=0}^{\tilde{i}-1} c^j(v), \quad \frac{dc^j(v)}{dv} > 0. \tag{46}$$

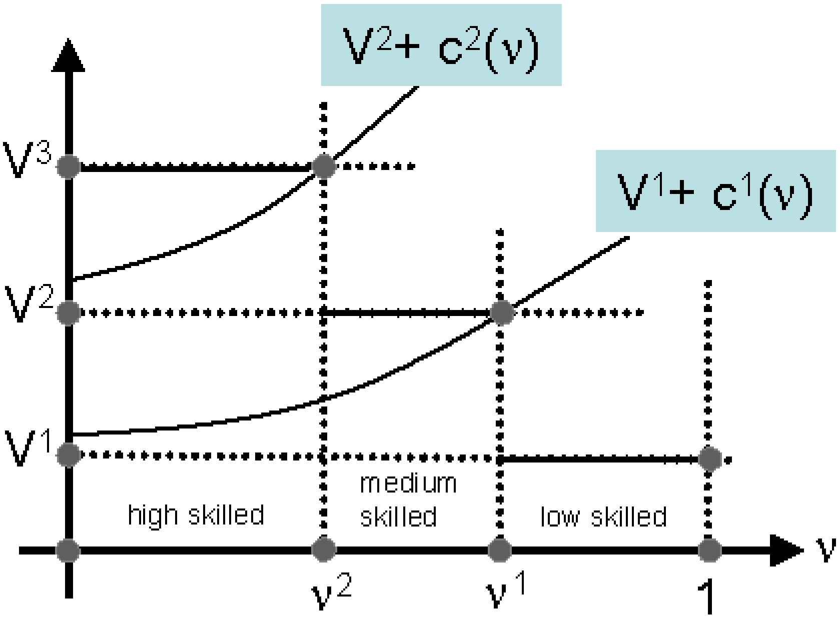
We assume heterogeneity of agents w.r.t. ability which is inversely related to the index $v \in [0,1]$. The *incremental* effort cost $c^i(v)$ of upgrading from skill $i-1$ to skill i is strictly increasing in v, i.e. $dc^i(v)/dv > 0$. Since $v = 0$ is the highest and $v = 1$ is the lowest ability, the assumption implies that the incremental effort cost to obtain the next higher degree increases with declining ability. We assume that ability of new agents is distributed according to the cumulative distribution function $\Gamma^D(\nu)$.

The discrete skill choice is most easily explained by starting with the lowest skill class $i = 1$ and asking whether it pays to obtain the next higher degree. Following this procedure, an agent with skill $i-1$ acquires the incremental education to obtain degree i if the following inequality derived from (46) holds: $V^{i,1}_{t,t} - c^{i-1}(v) > V^{i-1,1}_{t,t} \iff V^{i,1}_{t,t} > V^{i-1,1}_{t,t} + c^{i-1}(v)$. For high cost (low ability) persons, it is

[22] We abstract from all material start-up educational costs such as teacher salaries, books, tuition fees etc.

[23] Life-time utility net of educational effort would be V^1, $V^2 - c^1$ and $V^3 - (c^1 + c^2)$ for a low-, medium- and high-skilled agent where effort cost c^0 is normalized to zero, for convenience.

Figure 1: Discrete Education Choice



not worthwhile to incur the incremental educational costs. Hence, there exists a critical agent ν^{i-1} who is just indifferent between degrees $i-1$ and i:

$$V_{t,t}^{i,1} = V_{t,t}^{i-1,1} + c^{i-1}(\nu) \quad \Rightarrow \quad \nu_t^{i-1}. \tag{47}$$

Since educational costs increase in v, agents with *lower* ability $v > \nu^i$ and therefore higher costs are assigned to skill group $i-1$ as Figure 1 illustrates. Of all $N_{t,t}^1$ new agents entering the model with an age of 15 in period t, a share k_t^i chooses skill group i:

$$N_{t,t}^{i,1} = k_t^i \cdot N_{t,t}^1, \quad k_t^i = \begin{cases} 1-\Gamma^D\left(\nu_t^1\right) & : i=1, \\ \Gamma^D\left(\nu_t^1\right) - \Gamma^D\left(\nu_t^2\right) & : i=2, \\ \Gamma^D\left(\nu_t^2\right) & : i=3. \end{cases} \tag{48}$$

We assume, that the educational choice is open only to new households while older ones are locked into their previously chosen skill group. Hence, the law of motion for the number of individuals in age-group 1 with skill level i is

$$N_{t+1}^{i,1} = \gamma^1 \omega^1 N_t^{i,1} + N_{t+1,t+1}^{i,1} + netmig_{t+1}^{i,1}, \tag{49}$$

where $netmig_{t+1}^{i,1}$ is the number of net migrants into skill group i and the youngest age group 1. We specify the cost of education to be

$$c^i(v) = \left(\xi^{D,i}(1-v)\right)^{1/\varepsilon_D} \quad \Rightarrow \quad \nu_t^i = 1 - \left(\Delta_{t,t}^{D,i+1}\right)^{\varepsilon_D} / \xi^{D,i}, \tag{50}$$

where $\Delta^{D,i} \equiv V^i - V^{i-1}$, $\xi^{D,i} > 0$, $\varepsilon_D < 0$. If life-time utility of skill $i+1$ increases relative to i, the critical value ν^i rises and the share of skill group $i+1$ in the new cohort rises. For example, if the horizontal line V^3 in Figure 1 shifts up while $V^2 + c^2$ remains constant, the intersection point with the upward sloping curve $V^2 + c^2$ occurs at a higher value of ν^2, i.e. the critical ability ν^2 increases.

Remark 8 *The endogenous skill choice can be 'switched off' in the model simulations, so that the effects of a policy reform can also be analysed for an exogenously given distribution of educational levels.*

4 Demand for varieties of goods

The model allows for monopolistic competition implying cost minimization of the consumption basket on the household side. Cost minimization occurs in a first step with respect to the country of origin of goods and services and in a second step with respect to the firms producing differentiated goods.

4.1 Optimal division of the consumption bundle with respect to county of origin

In this section we derive the optimal division of the consumption bundle C^a into home-produced goods, $c^{h,a}$ and foreign goods, $c^{f,a}$. We assume again a CES-utility function of these two types of goods. The household minimizes the costs of the consumption bundle C^a by choosing goods produced at home and goods produced outside the economy in the rest of the world. The problem is given by:

$$\begin{aligned} pc &= \min_{c^h,c^f} p^{ch}c^h + p^{cf}c^f \\ s.t. \quad U_t\left(c^h,c^f\right) &= \Bigg[\underbrace{a_{cc}^{\frac{1}{p_{cc}}}\left(c^h\right)^{\frac{p_{cc}-1}{p_{cc}}} + (1-a_{cc})^{\frac{1}{p_{cc}}}\left(c^f\right)^{\frac{p_{cc}-1}{p_{cc}}}}_{x}\Bigg]^{\frac{p_{cc}}{p_{cc}-1}} \geq 1. \end{aligned}$$

Proposition 9 *Minimization of the price index pc leads to the following division of consumption in domestic and foreign produced goods and price index:*

$$\begin{aligned} c^{h,a} &= a_{cc}\left(\frac{pc}{p^{ch}}\right)^{p_{cc}} C^a, \\ c^{f,a} &= (1-a_{cc})\left(\frac{pc}{p^{cf}}\right)^{p_{cc}} C^a, \\ pc &= \left[a_{cc}\left(p^{ch}\right)^{1-p_{cc}} + (1-a_{cc})\left(p^{cf}\right)^{1-p_{cc}}\right]^{\frac{1}{1-p_{cc}}}. \end{aligned}$$

Proof. analogous to 13.1. ■

4.2 Optimal division of the home consumption bundle upon differentiated goods

The household again minimizes the price index across all home firms using a CES-utility function. For private households the number of domestic firms N^F is given. The problem is defined as:

$$\begin{aligned} p^{ch} &= \min_{\bar{c}} \sum_{j=1}^{N^F} p_j c_j \qquad s.t. \quad U(\bar{c}) = \left(\sum_{i=1}^{N^F} c_j^{\frac{1}{\eta}}\right) \geq 1, \\ \Rightarrow \frac{p_k}{p_j} &= \frac{c_k^{\frac{1-\eta}{\eta}}}{c_j^{\frac{1-\eta}{\eta}}} \Rightarrow c_k = \left(\frac{p_k}{p_j}\right)^{\frac{\eta}{1-\eta}} c_j. \end{aligned}$$

Let k equal the last firm N^F then it follows:

$$\sum_{i=1}^{N^F-1}\left(\frac{p_j}{p_{N^F}}\right)^{\frac{1}{1-\eta}} c_{N^F}^{\frac{1}{\eta}} + c_{N^F}^{\frac{1}{\eta}} = 1$$

$$\Rightarrow c_{N^F}^{\frac{1}{\eta}}\left[\sum_{j=1}^{N^F-1}\left(\frac{p_j}{p_{N^F}}\right)^{\frac{1}{1-\eta}} + 1\right] = 1$$

$$\Rightarrow c_{N^F} = \left[\frac{1}{\left(\frac{1}{p_{N^F}}\right)^{\frac{1}{1-\eta}}\left[\sum_{i=1}^{N^F} p_j^{\frac{1}{1-\eta}}\right]}\right]^{\eta} \Rightarrow c_j = \frac{p_j^{\frac{\eta}{1-\eta}}}{\left[\sum_{l=1}^{N^F} p_l^{\frac{1}{1-\eta}}\right]^{\eta}}.$$

Let $V = \left[\sum_{l=1}^{N^F} p_l^{\frac{1}{1-\eta}}\right]^{\eta}$. This results in:

$$p^{ch} = \sum_{l=1}^{N^F} p_l c_l = \frac{1}{V}\left[\sum_{l=1}^{N^F} p_l^{\frac{1}{1-\eta}}\right] = V^{\frac{1-\eta}{\eta}}.$$

Given $p = p_j = p_l \quad \forall j, l$ it follows that:

$$p^{ch} = \left(p^{\frac{1}{1-\eta}} N^F\right)^{1-\eta} = pN^{F^{1-\eta}} = pN^{F^{\frac{1}{1-\sigma}}}, \qquad \text{where } \eta = \frac{\sigma}{\sigma - 1}.$$

Using that $N^F = \left(\frac{p^{ch}}{p_j}\right)^{1-\sigma}$ and $p^{ch} = N^F p_j c_j$ gives that $c_j = \left(\frac{p^{ch}}{p_j}\right)^{\sigma}$, where c_j is the demanded amount from firm j given utility equals 1. One can conclude that:

$$\begin{aligned} C_j^{h,a} &= c_j c^{h,a} = \left(\frac{p^{ch}}{p_j}\right)^{\sigma} c^{h,a}, \\ C^{h,a} &= N^F c_j c^{h,a} = N^F\left(\frac{p^{ch}}{p_j}\right)^{\sigma} c^{h,a}. \end{aligned}$$

The demand for foreign varieties is determined in the same way with a different number of firms.

4.3 Demand from the Rest of the World

Domestic firms export goods in all other regions. It is assumed that firms are not able to discriminate between different buyers. Export demand functions for all possible regions may be rationalized by the following utility maximization problem of foreign agents:

$$\max_{e} u(e) + X \qquad s.t. \quad p^e e + X \leq Y^*,$$

where X is a non-traded good and e a traded goods aggregate composed of varieties from all over the world. Given this kind of preferences income effects are absorbed by demand for non-traded goods while demand for traded goods aggregate is a falling function $e(p^e)$ of its price-index. With a small share in foreign demand, domestic prices have a negligible influence on the price index p^e and leave foreign demand for the traded goods aggregate e largely unaffected. However, home prices affect derived demand for individual varieties. Foreign customers substitute toward home goods as these become cheaper. Exports involve *real trade costs* of the iceberg type. Only a fraction of $(1-\tau^e)$ of exported output arrives at foreign customers, who pay nevertheless p^e. The demand function is given by:

$$e^{RW} = e^0\left[\frac{1}{(1-\tau^e)p^e}\right]^{p_e} = e^0 (P^e)^{-p_e},$$

where p_e is the Armington elasticity and $(1-\tau^e)\,p^e = P^e$. The formula is the same as for consumption of households, with the exception that the price index of the foreign country equals 1 (normalization). The demand for goods for domestic firm j is given by:

$$e^{RW,j} = \left(\frac{P^e}{P^{e,j}}\right)^{\sigma} \Rightarrow EX^j = e^{RW,j} e^{RW} = \left(\frac{P^e}{P^{e,j}}\right)^{\sigma} e^{RW},$$
$$EX = N^F e^{RW,j} e^{RW} = N^F \left(\frac{P^e}{P^{e,i}}\right)^{\sigma} e^0 \left(P^e\right)^{-p_e}.$$

4.4 Government demand

The problem of the government choosing the optimal bundle is the same as for households. For this reason only the results are written here. The government minimizes the following:

$$\begin{aligned} p^{cg} &= \min_{cg^h, cg^f} p^{cg,h} cg^h + p^{cg,f} cg^f \\ s.t. \quad Ug\left(cg^h, cg^f\right) &= \left[a_{cg}^{\frac{1}{p_{cg}}} \left(cg^h\right)^{\frac{p_{cg}-1}{p_{cg}}} + \left(1-a_{cg}\right)^{\frac{1}{p_{cg}}} \left(cg^f\right)^{\frac{p_{cg}-1}{p_{cg}}}\right]^{\frac{p_{cg}}{p_{cg}-1}} \geq 1. \end{aligned}$$

This problem leads to:

$$\begin{aligned} cg^h &= a_{cg} \left(\frac{p^{cg}}{p^{cg,h}}\right)^{p_{cg}} \bar{CG}, \\ cg^f &= \left(1-a_{cg}\right) \left(\frac{p^{cg}}{p^{cg,f}}\right)^{p_{cg}} \bar{CG}, \\ p^{cg} &= \left[a_{cg} \left(p^{cg,h}\right)^{1-p_{cg}} + \left(1-a_{cg}\right) \left(p^{cg,f}\right)^{1-p_{cg}}\right]^{\frac{1}{1-p_{cg}}}, \\ CG_j^h &= cg_j^h cg^h = \left(\frac{p^{cg,h}}{p_j}\right)^{\sigma} cg^h, \\ CG^h &= N^F cg_j^h cg^h = N^F \left(\frac{p^{cg,h}}{p_j}\right)^{\sigma} cg^h, \\ p^{cg,h} &= p(=p_j) N^{F\,\frac{1}{1-\sigma}}. \end{aligned}$$

5 Aggregation

The simplicity and tractability of the perpetual youth model with a constant mortality rate rests on the fact that it allows for analytical aggregation. The same applies to the PA model. The advantage of approximating actual demographic and life-cycle properties with a low dimensional system is possible only if age groups can be aggregated analytically. In the PA model, agents are uniquely identified by their biography α. For all static variables, such as consumption, aggregate values per age group are given by

$$C_t^a = \sum_{\alpha \in \mathcal{N}_t^a} C_{\alpha,t}^a N_{\alpha,t}^a.$$

The aggregation of dynamic variables is somewhat complicated. However, aggregation of transfer wealth is particularly simple since individual variables are the same for all biographies within one age group. Aggregate transfer wealth is given by

$$T_t^a = \sum_{\alpha \in \mathcal{N}_t^a} T_{\alpha,t}^a N_{\alpha,t}^a,$$

where $T^a_{\alpha,t}$ is derived in (41e) for the workers, for example. In contrast to that, the dynamics of aggregate assets, pension- and productivity stocks is more difficult. This stems from the fact that, even within a particular age- and skill group, these values are different for different biographies α. The derivation is described in following subsections.

5.1 Skill Accumulation

The aggregate of the individual productivity of group a, $\Theta^{H,a}_t$, is

$$\Theta^{H,a}_t \equiv \sum\nolimits_{\alpha\in\mathcal{N}^a_t} \theta^{H,a}_{\alpha,t} N^a_{\alpha,t}. \tag{51}$$

Consider the law of motion for aggregate productivity Θ^H. By definition, productivity units $\Theta^{H,a}_{t+1}$ of group a at time $t+1$ are composed of productivity units of all those who already belonged to group a in period t and did not change the age group, plus productivity units of those who belonged to group $a-1$ in t, but aged at the end of period t and got their biography updated from α to α'. (The amendments to these equations for the implementation of migration in PuMA are presented at the end of this subsection.)

$$\Theta^{H,a}_{t+1} = \sum\nolimits_{\alpha\in\mathcal{N}^a_t} \theta^{H,a}_{\alpha,t+1} N^a_{\alpha,t+1} + \sum\nolimits_{\alpha\in\mathcal{N}^{a-1}_t} \theta^{H,a}_{\alpha',t+1} N^a_{\alpha',t+1}, \quad \alpha' = (\alpha, t+1)\,. \tag{52}$$

To obtain the law of motion, multiply this equation by G and use $N^a_{\alpha,t+1} = \gamma^a\omega^a N^a_{\alpha,t}$ and $G\theta^{H,a}_{\alpha,t+1} = \left[\left(1-\delta^{H,a}\right) + F^a_t\right]\cdot\theta^{H,a}_{\alpha,t}$, from (32). It is now evident why the symmetry in training effort e^a_t is important for aggregation. Otherwise, human capital production F^a_t would depend on the biography and aggregation would be impossible in the PA model. In the second term, note that by (2) the mass of movers with biographies $\alpha' = (\alpha, t+1)$ is a fraction $\gamma^{a-1}\left(1-\omega^{a-1}\right)$ of all $\alpha\in\mathcal{N}^{a-1}_t$. We thus substitute $N^a_{\alpha',t+1} = \gamma^{a-1}\left(1-\omega^{a-1}\right) N^{a-1}_{\alpha,t}$ as well as $G\theta^{H,a}_{\alpha',t+1} = \left(\left(1-\delta^{H,a-1}\right) + F^{a-1}_t\right)\theta^{H,a-1}_{\alpha,t}$ from (32), which yields

$$\begin{aligned}
a>1: \quad G\Theta^{H,a}_{t+1} &= \gamma^a\omega^a\left[\left(1-\delta^{H,a}\right)+F^a_t\right]\Theta^{H,a}_t \\
&+ \gamma^{a-1}\left(1-\omega^{a-1}\right)\left[\left(1-\delta^{H,a-1}\right)+F^{a-1}_t\right]\Theta^{H,a-1}_t, \\
a=1: \quad G\Theta^{H,1}_{t+1} &= \gamma^1\omega^1\left[\left(1-\delta^{H,1}\right)+F^1_t\right]\Theta^{H,1}_t + \theta^H_0 N^1_{t+1,t+1}.
\end{aligned}$$

In the first age group, the inflow of newborns is $N^1_{t+1,t+1}$. Each one is endowed with an exogenously given amount of productivity $\theta^{1,H}_{t+1,t+1} = \theta^H_0$.

Furthermore, migrants add to the aggregate productivity stock of group a. A number of $netmig^a_{t+1}$ migrants transfer their individual productivity to the host country. We assume that their individual productivity stock amounts to a share $\theta^{H,adj,a}$ of respective 'insiders'. This implies that the amount described above is increased by a factor $\theta^{H,adj,a}\frac{netmig^a_{t+1}}{N^a_{t+1}-netmig^a_{t+1}}$. Thus, migrants add to the aggregate productivity stock of different groups, which implies that

$$\begin{aligned}
a>1: \quad G\Theta^{H,a}_{t+1} &= \left[1+\theta^{H,adj,a}\frac{netmig^a_{t+1}}{N^a_{t+1}-netmig^a_{t+1}}\right]* \\
&\left[\gamma^a\omega^a\left[\left(1-\delta^{H,a}\right)+F^a_t\right]\Theta^{H,a}_t + \gamma^{a-1}\left(1-\omega^{a-1}\right)\left[\left(1-\delta^{H,a-1}\right)+F^{a-1}_t\right]\Theta^{H,a-1}_t\right], \\
a=1: \quad G\Theta^{H,1}_{t+1} &= \left[1+\theta^{H,adj,1}\frac{netmig^1_{t+1}}{N^1_{t+1}-netmig^1_{t+1}}\right]*\left[\gamma^1\omega^1\left[\left(1-\delta^{H,1}\right)+F^1_t\right]\Theta^{H,1}_t + \theta^H_0 N^1_{t+1,t+1}\right].
\end{aligned} \tag{53}$$

5.2 Pension Wealth

In general, the pension assessment base of an individual can be written as:

$$GP^a_{\alpha,t+1} = R^{P,a}\left[M^a + X^a P^a_{\alpha,t}\right].$$

In this equation, the variables for the different age groups are given by

$$\begin{aligned}
(W): X^a &= 1,\\
(M): X^a &= 1+\sigma^{P,a},\\
(R): X^a &= 1,\\
(R): M^a &= 0,\\
(W): M^a &= \bar{\delta}^a\delta^a\left[(1-u^a+u^a b_1)\,m^a w^a\theta^a_\alpha l^a + m^a_1\right] + \left(1-\bar{\delta}^a\right)X^a_0,\\
(M): M^a &= \bar{\delta}^a\sigma^M\left(\delta^a\right)\left[(1-u^a+u^a b_1)\,m^a w^a\theta^a_\alpha l^a + m^a_1\right] + \left(1-\bar{\delta}^a\right)X^a_0,
\end{aligned}$$

where (R) stands for retiree age groups. Multiplying by $\omega^a\gamma^a N^a_{\alpha,t}$ and summing over α yields:

$$G\omega^a\gamma^a\sum_{\alpha\in N^a_t} N^a_{\alpha,t}P^a_{\alpha,t+1} = \omega^a\gamma^a\sum_{\alpha\in N^a_t} N^a_{\alpha,t}R^{P,a}\left(M^a_t + X^a P^a_{\alpha,t}\right)$$

and finally results in:

$$\begin{aligned}
GP^a_{t+1} &= \omega^a\gamma^a R^{P,a}\left(N^a_t M^a_t + X^a P^a_t\right) + \\
&\quad +\gamma^{a-1}\left(1-\omega^{a-1}\right)R^{P,a-1}\left(N^{a-1}_t M^{a-1}_t + X^{a-1}P^{a-1}_t\right).
\end{aligned}$$

Analogously to individual productivity stocks, migrants transfer pension claims to the destination country, amounting to a share $P^{adj,a}$ of respective 'insiders'. In analogy to the aggregate productivity stock, this implies:

$$\begin{aligned}
GP^a_{t+1} &= \left[1 + P^{adj,a}\frac{netmig^a_{t+1}}{N^a_{t+1} - netmig^a_{t+1}}\right] * \\
&\quad \left[\omega^a\gamma^a R^{P,a}\left(N^a_t M^a_t + X^a P^a_t\right) + \gamma^{a-1}\left(1-\omega^{a-1}\right)R^{P,a-1}\left(N^{a-1}_t M^{a-1}_t + X^{a-1}P^{a-1}_t\right)\right].
\end{aligned} \tag{54}$$

However, pension claims that have been acquired until the time of migration are paid by the country of origin, while pension claims acquired after migration are paid by the destination country. Nevertheless, claims to foreign public pension systems are transferred to domestic claims, see 55. The domestic public pension system is compensated by transfers from foreign governments $ex^{Pens,Corr}$. To keep track of this, we perform an adjustment in the public sector. We introduce a stock of net international government pension claims $P^{Gov,a}_t$, which follows the law of motion:

$$GP^{gov,a}_{t+1} = \omega^a\gamma^a R^{P,a}X^a P^{gov,a}_t + \gamma^{a-1}\left(1-\omega^{a-1}\right)R^{P,a-1}X^{a-1}P^{gov,a-1}_t \tag{55}$$

$$-netmig^a_{t+1}P^{adj,a}\frac{GP^a_{t+1}}{N^a_{t+1}}. \tag{56}$$

To illustrate: in case of positive net migration to the modelled country, this stock $P^{Gov,a}$ is negative. In that case, home government's pension expenditures are reduced by $ex^{Pens,Corr}$ because foreign governments pay for this part.

From

$$S_\alpha^a = \tilde{\lambda}^a P_\alpha^a$$

and from the property that $\tilde{\lambda}_\alpha^a = \tilde{\lambda}^a$ is identical for all biographies, we get aggregate pension wealth for an age group:

$$S^a = \sum_\alpha S_\alpha^a N_\alpha^a = \sum_\alpha \tilde{\lambda}^a P_\alpha^a N_\alpha^a = \tilde{\lambda}^a P^a. \tag{57}$$

5.3 Assets

The law of motion for aggregated financial assets of the households of an age group is given by the following equations.[24] Note that migration adds a similar term to this law of motion as for skill accumulation and pension wealth.

$$GA_{t+1}^1 = \left[1 + A^{adj,1} \frac{netmig_{t+1}^1}{N_{t+1}^1 - netmig_{t+1}^1}\right] R_{t+1}^\tau \omega^1 Sav_t^1, \tag{58a}$$

$$GA_{t+1}^a = \left[1 + A^{adj,a} \frac{netmig_{t+1}^a}{N_{t+1}^a - netmig_{t+1}^a}\right] R_{t+1}^\tau \left[\omega^a Sav_t^a + \left(1 - \omega^{a-1}\right) Sav_t^{a-1}\right] \quad a = 2, ..., A, \tag{58b}$$

where

$$Sav^a = A^a + (y^a + z^a + iv^a)\, N^a - pc^a C^a.$$

[24] For the derivation of these equations see Grafenhofer et al. (2007).

6 Production and Wage Bargaining

Firms in the economy are divided in *capital firms*, transforming final goods in capital goods, and *final goods firms*, producing goods for private and public consumption, export and investment. Final goods producers operate in a monopolistic competition environment, capital firms in a perfectly competitive environment. Final goods firm j produces output $Y_{t,j}$ by renting or buying four kinds of input, capital K_t and three different types of labour input (skills) $L^D_{t,i}$. Output of final goods firm j is produced via a linear homogeneous nested CES-production function:

$$Y_{t,j} = F^Y(u_{t,j}K_{t,j}, L^D_{t,j,1}, L^D_{t,j,2}, L^D_{t,j,3}), \tag{59}$$

where $u_{t,j}$ is the level of utilization of the capital stock. The higher $u_{t,j}$, the higher the utilization of the capital stock and level of production at costs of higher depreciation. Perfect competition in the capital sector implies that the required rate of return of investment is given by the internationally determined interest rate. Technological progress is labour augmented and exogenously given in the model. Thus, there is no endogenously determined steady state growth. The different types of labour input are imperfect substitutes. We assume that high-skilled labour and capital are more complementary than unskilled labour and capital. The first empirical evidence is presented in Griliches (1969) who finds that capital and skilled labour inputs are more complementary than capital and unskilled labour. Duffy et al. (2004) find evidence in support of complementarity, if the defined threshold for skill disjunction is set low, as well as support for a CES production function. Therefore, we use a three-level CES form and set the threshold for low-skilled workers rather low. Goldin and Katz (1998) argue that capital-skill complementarity may be a transitory phenomenon. Krusell et al. (2000) find instead that capital-skill complementarity and changes in inputs in the aggregate production function can explain most of the variations in the skill premium.

6.1 Job Matching

PuMA is based on search unemployment as pioneered by Mortensen (1986). In the model separate labour markets for each age and skill group are implemented. From the active population $\delta^a_i \bar{\delta}^a_i N^a_i$, a share ε^a_i is hired immediately, while the remaining $(1-\varepsilon^a_i)\,\delta^a_i \bar{\delta}^a_i N^a_i$ have to search for a job. An individual of age group a and skill group i has a search effort of s^a_i. The number of total search units per age and skill group is $S^{M,a}_i = s^a_i\,(1-\varepsilon^a_i)\,\delta^a_i \bar{\delta}^a_i N^a_i$, which determines, together with the sum of the number of vacancies $\sum_j v^a_{j,i}$ posted across all firms j, the number of matches $M^a_i\left(S^{M,a}_i \sum_j v^a_{j,i}\right)$ between individuals looking for a job and vacancies. Given matching frictions, only a share f^a_i of search units is matched and a share q^a_i of vacancies is filled:

$$f^a_i S^{M,a}_i = M^a_i\left(S^{M,a}_i, \sum_j v^a_{j,i}\right) = q^a_i \sum_j v^a_{j,i}, \quad S^{M,a}_i \equiv s^a_i\,(1-\varepsilon^a_i)\,\delta^a_i \bar{\delta}^a_i N^a_i, \quad \Theta^{M,a}_i \equiv \frac{\sum_j v^a_{j,i}}{S^{M,a}_i}, \tag{60}$$

where $\Theta^{M,a}_i$ denotes labour market tightness. A fraction $f^a_i\left(\Theta^{M,a}_i\right)$ of all search units and a fraction $q^a\left(\Theta^{M,a}_i\right)$ of all vacancies are successfully matched. Assuming $M^a_i(\cdot)$ to be quasiconcave and linear homogeneous implies $f'\left(\Theta^M\right) > 0 > q'\left(\Theta^M\right)$ and $q\left(\Theta^M\right) = \Theta^M f\left(\Theta^M\right)$.[25]

Given matching frictions, the firms hire

$$L_i^{H,a} = \varepsilon_i^a \delta_i^a \bar{\delta}_i^a N_i^a + M_i^a \left(S_i^{M,a}, \sum_j v_{j,i}^a \right) \tag{61}$$

by posting $\sum_j v_{j,i}^a$ vacancies. As all final goods firms are charging the same price and produce varieties they are all of the same size, implying that $L_{j,i}^{H,a} = L_i^{H,a}/N^F$. After hiring, a worker is kept with probability $p_{man,j,i}^a$. This probability depends on the firm's managerial effort to provide a productive match. We assume that more managerial effort raises the success probability p_{man} one to one, but at convex increasing costs $\varphi^F(p_{man})$ per worker, giving raise to total costs $\sum_{i,a} \varphi^F\left(p_{man,j,i}^a\right) \cdot L_{j,i}^{H,a}$ for final goods firm j. We assume that these are real costs reducing produced output. Final goods firms can increase productivity of their employees by investing in firm-sponsored training $e_j^{F,a}$. Providing $e_j^{F,a}$ units of training raises productivity by $\theta^{F,a}\left(e_j^{F,a}\right)$. However, firm-sponsored training implies costs $\varphi^{FS}\left(e_j^{F,a}\right)$ per hired worker, giving raise to total costs $\varphi^F(e) \cdot L^H$. Total productivity of an individual in the job is then given by $\theta_{\alpha,t}^a = \theta_{\alpha,t}^{H,a} \cdot \theta_{t,j}^{F,a}$.

A worker of age group a and skill group i effectively supplies $l_i^a \theta_{i,\alpha}^a$ of labour input, so that effective labour input of firm j per age- and skill-group is given by

$$L_{j,i}^{D,a} = l_i^a \bar{\theta}_{j,i}^a p_{man,j,i}^a L_{j,i}^{H,a}. \tag{62}$$

Summing up, we get $L_i^D = \sum_a L_i^{D,a}$ units of input per skill group and $L^D = \sum_i L_i^D$ total labour input.

6.2 Capital goods firm

The capital goods firm provides capital goods to final goods firms by transforming final goods into capital goods. It demands final goods in the Home Country and from the Rest of the World. The optimal division of the final goods is determined by cost minimization. The firm value is defined by the present value of the stream of dividends. Investment I causes installation costs J, measured in terms of output. $J(\cdot)$ guarantees that firms adjust their capital stock smoothly to the optimal level. Total investment costs $I + J$ consist of market spending and adjustment costs (deducted from output). Capital depreciates at a variable rate $\delta(u_t)$, depending on the level capital utilization, which is determined by the final goods firm. The evolution of the capital stock follows the following path

$$GK_{t+1} = (1 - \delta(u_t)) K_t + I_t. \tag{63}$$

The value function of the capital goods firm is given by, where K is the stock variable of the firm and χ dividend payments to households:

$$V^F(K_t) = \max_I \left\{ \chi_t^K + \frac{GV^F(K_{t+1})}{R_{t+1}} \right\}, \tag{64}$$

$$s.t. \; : \; GK_{t+1} = (1 - \delta(u_t)) K_t + I_t, \tag{65}$$

$$\text{where } R_{t+1} = 1 + r_{t+1}. \tag{66}$$

[25] For the functional form as described in Section 10 this implies $M = M_0 \cdot \left(L^M\right)^\sigma \sum_j v_j^{1-\sigma}$, we get $f = M_0 \left(\Theta^M\right)^{1-\sigma}$, $q = M_0 / \left(\Theta^M\right)^\sigma$ and $f = \Theta^M \cdot q$.

Dividends of capital firms are:

$$\chi_t^K = \left(1-t^{prof}\right)\left[p_t^k K_t - p_t^{inv} J_t + p_t^{inv} K_t \left(\delta\left(u_t\right) - \bar{\delta}^K\right) - t^{cap} p_t^{inv} K_t\right] + \quad (67)$$

$$+t^{prof}\left(\delta\left(u_t\right) p_t^{inv} K_t + sub^I p_t^{inv} I_t\right) - p_t^{inv} I_t, \quad (68)$$

where p^k is the rental price of capital and p^{inv} the price index for investment goods. As the capital firm cannot control capital utilization, but has to bear higher (lower) costs of by higher (lower) deprecation, we assume that production firms compensate the capital firm by a payment (refund) to the higher (lower) than normal depreciation rate. The variable sub^I reflects tax allowances for investment and t^{cap} is a tax rate on the capital stock of firms.The capital goods firms maximize profits with respect to the level of investment today. The first order condition and the envelope condition are given by:

$$I : \frac{tob_{t+1}}{R_{t+1}} = p^{inv}\left[1 + \left(1-t^{prof}\right) J_I - t^{prof} sub^I\right], \quad (69)$$

$$K : tob_t = \left(1-t^{prof}\right)\left(p^k + p^{inv}\left(\delta\left(u\right) - \bar{\delta}^K\right) - p^{inv} J_K - p^{inv} t^{cap}\right) + t^{prof} p^{inv} \delta\left(u\right) + \quad (70)$$

$$+\left(1-\delta\left(u\right)\right)\frac{tob_{t+1}}{R_{t+1}}, \quad (71)$$

where tob is Tobin's q.

The FOC with respect to investment (69) shows that firms invest as long as the discounted shadow value of an additional unit of investment equals the costs. The shadow value tob_t equals the discounted amount of dividend payments arising from an additional unit of capital. We specify the adjustment cost function in such a way that, $J = J_I = J_K = 0$ in the steady state. In a steady state, these two equations imply $tob = p^{inv}\left(1 - t^{prof} sub^I\right) R$ and

$$UC \equiv p^k = \frac{p^{inv}\left(1 - t^{prof} sub^I\right)\left(r + \delta\left(u\right)\right) - t^{prof} p^{inv} \delta\left(u\right)}{\left(1-t^{prof}\right)} + p^{inv} t^{cap} - p^{inv}\left(\delta\left(u\right) - \bar{\delta}^K\right),$$

where UC stands for user costs of capital in the steady state.

Proposition 10 *(Hayashi) The value of the capital firm is*

$$V^F = tob \cdot K. \quad (72)$$

Proof. Multiplication of the envelope condition (71) with K_t implies

$$tob{\cdot}K = \left(1-t^{prof}\right)\left(p^k K + p^{inv}\left(\delta\left(u\right) - \bar{\delta}^K\right) K - p^{inv} J_K K - p^{inv} t^{cap} K\right) + t^{prof} \delta\left(u\right) K + \frac{tob_{t+1}}{R_{t+1}}\left(1-\delta\left(u\right)\right) K.$$

Using equation (65) for the capital stock K and the Euler theorem for linear homogeneous functions implies

$$tob{\cdot}K = \left(1-t^{prof}\right)\left(p^k K + p^{inv}\left(\delta\left(u\right) - \bar{\delta}^K\right) - p^{inv} J_K K - p^{inv} t^{cap} K\right) + t^{prof} \delta\left(u\right) K - \frac{tob_{t+1}}{R_{t+1}} I + \frac{tob_{t+1}}{R_{t+1}} G K_{t+1}.$$

Using the FOC for investment, (69),we get

$$tob \cdot K = \chi^K + \frac{tob_{t+1}}{R_{t+1}} G K_{t+1},$$

■

Finally, the optimal investment decision of the firm is obtained from the first order condition (69). This condition relates optimal investment to the firm value V^K, to the capital stock K, and to other parameters like the depreciation rate of capital $\delta(u)$, the growth rate g and the scale parameter for the adjustment costs ψ (for the functional form of the adjustment costs, see the Appendix (equation (107)).

Proposition 11 *Optimal investment of the firm is given by the positive root of the solution of the quadratic equation* $I^2 + aI - b = 0$,

$$I = \frac{1}{2}\left(-a + \sqrt{a^2 + 4b}\right), \tag{73}$$

where

$$\begin{aligned} a &= \left[(1-\delta(u)) - (\delta(u)+g) + \frac{1 - t^{prof} sub^I}{(1-t^{prof})\psi}\right] K, \\ b &= -\left\{\left[(1-\delta(u))\left(1-t^{prof} sub^I\right) - (1-\delta(u))(\delta(u)+g)\left(1-t^{prof}\right)\psi\right] K - \frac{GV_{t+1}^K}{p^{inv} R_{t+1}}\right\} \frac{K}{(1-t^{prof})\psi}. \end{aligned}$$

Proof. See Appendix 13.6 ■

The investment goods bundle consists of imported and home produced goods and of corresponding varieties. The capital goods firm has the following cost minimization problem:

$$\begin{aligned} p^{inv} &= \min_{i^h, i^f} p^{inv,h} i^h + p^{inv,f} i^f \\ s.t. \quad Ui\left(i^h, i^f\right) &= \left[a_i^{\frac{1}{p_i}} \left(i^h\right)^{\frac{p_i - 1}{p_i}} + (1-a_i)^{\frac{1}{p_i}} \left(i^f\right)^{\frac{p_i - 1}{p_i}}\right]^{\frac{p_i}{p_i - 1}} \geq 1. \end{aligned}$$

This problem leads to:

$$\begin{aligned} i^h &= a_i \left(\frac{p^{inv}}{p^{inv,h}}\right)^{p_i} I, \\ i^f &= (1-a_i)\left(\frac{p^{inv}}{p^{inv,f}}\right)^{p_i} I, \\ p^{inv} &= \left[a_i \left(p^{inv,h}\right)^{1-p_i} + (1-a_i)\left(p^{inv,f}\right)^{1-p_i}\right]^{\frac{1}{1-p_i}}, \\ i_j^h &= i_j^h i^h = \left(\frac{p^{inv,h}}{p_j}\right)^{\sigma} i^h, \\ i^h &= N^F i_j^h i^h = N^F \left(\frac{p^{inv,h}}{p_j}\right)^{\sigma} i^h, \\ p^{inv,h} &= p(=p_j) N^{F\frac{1}{1-\sigma}}. \end{aligned}$$

6.3 Final goods firms

Final goods firms produce output by means of capital and labour input. The final goods sector is characterized by free entry, however firms have to bear fixed costs in each period they are in the market. Competition between producers of varieties and the free entry condition leads to zero profit of final goods firms in equilibrium. This condition determines the number of domestic final goods firms N^F active in the market. They face a downward sloping demand curve in their own price p_j. These firms hire workers from the labour market, fire a share of the labour force and rent capital from the capital goods firms. Offering v_i^a vacancies implies vacancy costs $\kappa_i^a(v_i^a)$ to the firm, for the functional form see equation (108) in the Appendix. Firms are also assumed to incur fixed costs v_0^i that are independent of the number of vacancies they offer.

The wage bill of firm j is given by:

$$\left(1+t^{sscF}\right) wL_j^D = \sum_{a,i}\left[\left(1+t_i^{sscF,a}\right) w_i^a L_{j,i}^{D,a} + z_i^{F,a} p_{man,j,i}^a L_{j,i}^{H,a}\right]. \tag{74}$$

t^{sscF} represents social security contributions and other wage-dependent taxes and contributions of the employer. The variable $z_i^{F,a}$ includes flat social security contributions or taxes of employers that are not related to wages, such as in Denmark. The model includes firing costs incurred by firms when dismissing workers. Firing costs are assumed to consist of severance payments τ^S, firing taxes τ^{Fire} and administrative costs (like law suits; modelled as lost output) τ^C. In the following, severance payments and firing taxes are modelled similar, whereas administrative costs are deducted from output. For simplification we define $\tau^F = \tau^S + \tau^{Fire}$. Severance payments and firing taxes depend on the wage level and number of hours worked. This is not the case for administrative costs. In addition final goods firms bear labour adjustment costs. It is assumed that adjustment of employment compared to the previous period leads to costs $Ladj_i$, in form of lost output. Firms do not take into account that the employment decision in period t has an impact on the following periods. Also new firms bear adjustment costs.

Firms have to pay taxes on profits t^{prof}. The assessment base is given by output deducted by capital costs, the wage-sum inclusive taxes and contributions paid by employers, the vacancy and managerial costs, firing taxes plus employment subsidies. For simplicity in the following we neglect index j of the firm:

$$\chi = \tag{75}$$
$$p\bar{Y} - p^k K - p^{inv} K\left(\delta(u) - \bar{\delta}^K\right) - \left(1+t^{sscF}\right) wL^D - T^F + \sum_{i,a}\left(sub^L p_{man,i}^a + sub^T e_i^{F,a}\right) L_i^{H,a} -$$
$$-\sum_{i,a} \tau_i^{F,a} fac_i^a \left(1-p_{man,i}^a\right) w_i^a l_i^a \bar{\theta}_i^a L_i^{H,a},$$
$$\bar{Y} = Y - FC - \sum_{i,a}\kappa_i^a(v_i^a) - \sum_i Ladj_i - \sum_{i,a}\left(\varphi_i^{FS,a}\left(e_i^{F,a}\right) + \varphi_i^{F,a}\left(p_{man,i}^a\right)\right) L_i^{H,a} -$$
$$-\sum_{i,a}\tau_i^{C,a} fac_i^a \left(1-p_{man,i}^a\right) L_i^{H,a} \tag{76}$$
$$T^F =$$
$$t^{prof}\left(p\bar{Y} - p^k K - p^{inv} K\left(\delta(u) - \bar{\delta}^K\right) - \left(1+t^{sscF}\right) wL^D + \sum_{i,a}\left(sub^L p_{man,i}^a + sub^T e_i^{F,a}\right) L_i^{H,a}\right)$$
$$+t^{prof}\left(-\sum_{i,a}\tau_i^{F,a} fac_i^a\left(1-p_{man,i}^a\right) w_i^a l_i^a \bar{\theta}_i^a L_i^{H,a}\right),$$

where $sub_i^{L,a}$ is a government employment subsidy and FC the fixed costs of the firm in terms of lost output. In addition, firms may get a subsidy of sub^T per unit of firm-sponsored training.

Remark 9 *The contributions of the firms to the social security system* t^{sscF} *are fully recognized as a tax. By contrast, the employees contributions are perceived only partly as taxes, because they generate future claims in the social security system.*

Firms maximize the dividend payment χ by optimally choosing the number of vacancies v_i^a, capital K, the firing rate $(1-p_{man,i}^a)$, the level of capital utilization u, and firm-sponsored training $e_i^{F,a}$. The problem of final goods firms is

$$\max_{v,K,p,e^F,u} \chi \text{ s.t.} \tag{77a}$$

$$\bar{Y} = D(p) \tag{77b}$$

$$D(p) = d_0 \cdot \left(\frac{p^H}{p}\right)^{\sigma}, \tag{77c}$$

where p^H is the price level of firms producing in the Home country and taken as given by a single firm, i.e. single firms cannot influence the average price level across all domestic firms p^H. Given p^H, the demand for goods in the home country is compromised in d_0. The optimality conditions are given by:[26]

$$v_i^a : q_i^a \left\{ \begin{array}{l} \bar{\lambda}\left(F_{L,i}^{Y,a} l_i^a \bar{\theta}_i^a p_{man,i}^a - \varphi_i^{F,a}\left(p_{man,i}^a\right) - \varphi_i^{FS,a}\left(e_i^{F,a}\right) - \tau_i^{C,a} fac_i^a\left(1-p_{man,i}^a\right) - Ladj_{N_{t,i}^W} p_{man,i}^a\right) + \\ + \left(1-t^{prof}\right)\left[-\left(1+t_i^{sscF,a}\right) p_{man,i}^a w_i^a \bar{\theta}_i^a l_i^a + \left(sub^L - z_i^{F,a}\right) p_{man,i}^a + sub^T e_i^{F,a}\right] + \\ + \left(1-t^{prof}\right)\left[-\tau_i^{F,a} fac_i^a \left(1-p_{man,i}^a\right) w_i^a l_i^a \bar{\theta}_i^a\right] \end{array} \right\} = \kappa_i^{a\prime}\bar{\lambda}, \tag{78}$$

$$p_{man,i}^a : \bar{\lambda}\left(F_{L,i}^{Y,a} l_i^a \bar{\theta}_i^a + \tau_i^{C,a} fac_i^a - Ladj_{N_{t,i}^W}\right) - \left(1-t^{prof}\right)\left\{\left(1+t_i^{sscF,a}\right) w_i^a l_i^a \bar{\theta}_i^a + z_i^{F,a}\right\} - \left(1-t^{prof}\right)\left\{-sub^L - \tau_i^{F,a} fac_i^a w_i^a l_i^a \bar{\theta}_i^a\right\} = \left(\varphi_i^F\left(p_{man,i}^a\right)\right)' \bar{\lambda}, \tag{79}$$

$$e_i^{F,a} : \left(\theta_i^{F,a}\left(e_i^{F,a}\right)\right)' l_i^a \theta_i^{H,a} \left[\begin{array}{l} p_{man,i}^a\left(\bar{\lambda} F_{L,i}^{Y,a} - \left(1-t^{prof}\right)\left(1+t_i^{sscF,a}\right) w_i^a\right) - \\ - \left(1-t^{prof}\right)\left(1-p_{man,i}^a\right)\tau_i^{F,a} fac_i^a w_i^a \end{array} \right] + \left(1-t^{prof}\right) sub^T = \left(\varphi_i^{FS}\left(e_i^{F,a}\right)\right)' \bar{\lambda}, \tag{80}$$

$$K : \left(p^k + p^{inv}\left(\delta(u) - \bar{\delta}^K\right)\right)\left(1-t^{prof}\right) = \bar{\lambda} F_K^Y, \tag{81}$$

$$u : p^{inv} K \delta'(u)\left(1-t^{prof}\right) = \bar{\lambda} F_u^Y, \tag{82}$$

$$\lambda : \bar{Y}\left(1-t^{prof}\right) = \lambda\sigma D(p)\frac{1}{p}. \tag{83}$$

The last equation implies that:

$$p = \lambda\sigma\frac{D(p)}{\bar{Y}\left(1-t^{prof}\right)} = \frac{\lambda\sigma}{1-t^{prof}} \Rightarrow \left(\left(1-t^{prof}\right)p - \lambda\right) = (\lambda\sigma - \lambda) = \bar{\lambda}$$

The firm's first order conditions (FOC) equate the marginal revenues and the marginal costs of providing one additional unit of vacancies, managerial effort, firm-sponsored training, capital and capital utilization. Equation (78) states that the expected marginal gain of a vacancy for the firm equals the marginal costs κ' of posting an additional vacancy. Ex ante, the firm keeps a matched worker with probability $p_{man,i}^a$,

[26] We assume that firms can not target their hiring and firing decisions to specific biographies, i.e. productivity levels. Thus, in the three first order conditions for hiring (78), firing (79) and training (80), firms base their decision on the average productivity level of an age and skill group. The assumption of symmetry of firms requires the assumption that the ex-post average productivity within a firm equals average productivity in the economy for each age- and skill-group.

thus generating labour productivity and paying labour costs. With probability $\left(1-p_{man,i}^{a}\right)$ the employee is fired again, which causes firing costs τ^{F} for the firm. Furthermore, the FOC for vacancies includes employment and training subsidies, hiring vouchers, flat social security contributions and the costs for managerial effort of the firm and firm-sponsored training. The condition may be rewritten as

$$\kappa_i^{a\prime} = \frac{q_i^a \bar{\theta}_i^a l_i^a}{\bar{\lambda}} \left[\left(1-t^{prof}\right)\left[p_{man,i}^a\left(1+t_i^{sscF,a}\right)+\tau_i^{F,a} fac_i^a\left(1-p_{man,i}^a\right)\right]\right]\left[w_+^a - w_i^a\right]; \quad (84)$$

$$w_+^a = \frac{\left\{\begin{array}{c}\bar{\lambda}\left[F_{L,i}^{Y,a} p_{man,i}^a - \frac{\varphi_i^{F,a}\left(p_{man,i}^a\right)+\varphi_i^{FS,a}\left(e_i^{F,a}\right)+\tau_i^{C,a} fac_i^a\left(1-p_{man,i}^a\right)+Ladj_{N_{t,i}^W} p_{man,i}^a}{\bar{\theta}_i^a l_i^a}\right]+ \\ +\left(1-t^{prof}\right)\frac{\left(sub^L - z_i^{F,a}\right)p_{man,i}^a + sub^T e_i^{F,a}}{\bar{\theta}_i^a l_i^a}\end{array}\right\}}{\left(1-t^{prof}\right)\left[p_{man,i}^a\left(1+t_i^{sscF,a}\right)+\tau_i^{F,a} fac_i^a\left(1-p_{man,i}^a\right)\right]},$$

so that the optimal number of vacancies is determined by the difference of the wage level w^a and the reservation wage of the firm w_+^a.

Equation (79) is the FOC with respect to the managerial effort and equates marginal effort costs with the additional gain from providing more effort. By increasing this effort, the firm keeps more workers and accumulates additional profit amounting to labour productivity minus labour costs. The firing decision is influenced by the public system design, as the firm avoids paying firing taxes and receives employment subsidies if it keeps the worker. The FOC with respect to firm-sponsored training is given by (80). The marginal increase of firm-specific productivity is given by $\left(\theta^{F,a}\left(e^{F,a}\right)\right)'$ and the marginal gain of an additional unit of training is determined by higher output, diminished by higher wage costs, higher potential firing costs for the firm and higher training costs, and can be incentivised by a training subsidy.

Equation (81) states that firms demand capital until the marginal product of capital F_K^Y equals the price for capital p^k plus the additional costs (gains) for more (less) then average utilisation of capital. The first order condition for capital utilisation (82) is very similar. It equates the marginal product of utilisation F_u^Y to the costs implied by higher utilisation. If firms use the capital stock more intensively (higher u), they have to pay the costs of higher depreciation $p^{inv} K \delta'(u)$ to capital firms.

6.4 Wage Bargaining

Once a match has been generated, the worker and the firm must agree upon the wage per efficiency unit; otherwise the match is not successful and the worker-firm-pair split. Each side gains a surplus that depends on the agreed wage and her fall-back position. Either side accepts a match whenever her surplus is positive. A higher wage raises the worker's surplus but reduces the gains of the firm.

The wage level in a firm[27] is given by maximizing the Nash-bargaining product (85) with respect to

[27] Wage bargaining occurs on the firm level. The assumption of symmetric firms requires the same wage level within each age- and skill-group across the firms. For this reason the wage bargaining power is the same across the firms.

the wage rate:

$$w^a = \arg\max_{w^a} \left(w_+^a - w^a\right)^{\xi} \left(w^a - w_-^a\right)^{1-\xi}, \tag{85}$$

$$w_+^a = \frac{\left\{ \bar{\lambda} \left[F_{L,i}^{Y,a} p_{man,i}^a - \frac{\varphi_i^{F,a}\left(p_{man,i}^a\right) + \varphi_i^{FS,a}\left(e_i^{F,a}\right) + \tau_i^{C,a} fac_i^a \left(1 - p_{man,i}^a\right) + Ladj_{N_{t,i}^W} p_{man,i}^a}{\bar{\theta}_i^a l_i^a} \right] + + \left(1 - t^{prof}\right) \frac{\left(sub^L - z_i^{F,a}\right) p_{man,i}^a + sub^T e_i^{F,a}}{\bar{\theta}_i^a l_i^a} \right\}}{\left(1 - t^{prof}\right) \left[p_i^a \left(1 + t_i^{sscF,a}\right) + \tau_i^{F,a} fac_i^a \left(1 - p_{man,i}^a\right) \right]}; \tag{86}$$

$$w_-^a = p_{man}^a \frac{\left(pc^a \left(\varphi^L + h_u^a\right) - z_w^a + \xi_1 z_u^a + \left(1 - \xi_1\right) b^{0,a} - \frac{\gamma^a e^a F_{E,\alpha}^a}{R^\tau} \frac{\bar{\chi}_{t+1}^a}{\bar{\eta}_{t+1}^a} \right)}{\left[p_{man}^a l^a \Gamma^a + l^a \tau^{S,a} fac^a \left(1 - t^{w,S,a}\right) \right]} \tag{87}$$

$$\Gamma^a = (1 - \tau)(1 - \xi_1 b) + (1 - b_1) m \frac{\gamma^a R^{P,a}}{R^\tau} \frac{\bar{\lambda}_{t+1}^a}{\bar{\eta}_{t+1}^a} \frac{\sigma^M}{\delta^a} - \tau^S \left(1 - t^{w,S}\right) fac, \tag{88}$$

$$\sigma^M / \delta^a = 1 \text{ except for the mixed group.}$$

The firm's surplus from employing a new worker with productivity $\theta_\alpha^{i,a}$ (see 84) is the product of two terms, where only $\left(w_+^a - w^a\right)$ includes the wage rate, so that the other term does not influence the wage bargaining outcome. The reservation wage w_+^a of the firm increases with labour productivity, employment and training subsidies, decreases with managerial and firm-specific training costs, and taxes (wage-related and flat) on labour paid by the firm. It also includes the potential of firing the worker. Analogously to the firm's surplus, the surplus of the employee can be written as the product of two terms, where only $\left(w^a - w_-^a\right)$ includes the wage, see (37). The reservation wage of the worker w_-^a increases with the effort costs φ^L and the value of home production h_u^a, and decreases with the future increase in productivity as additional productivity is only acquired when employed. Furthermore, it depends on the public transfers, labour taxes and severance payments. At the moment of wage bargaining, search and vacancy costs as well as managerial effort and firm-sponsored training costs are sunk costs, so that they don't influence the wage bargaining process.

Differentiating the Nash bargaining product (85) with respect to the wage gives the wage as a weighted sum of the firm and worker reservation wages. Higher reservation wages of firms and employees increase the wage.

Proposition 12 *The wage is given by*

$$w^a = (1 - \xi) w_+^a + \xi w_-^a. \tag{89}$$

7 Public Sector

Concerning the public sector, there are separate budgets for the general government and the social security system.[28] In the *social security budget*, the public sector collects contributions from employers and employees, and spends for pensions and unemployment benefits. This budget may also receive transfers Tr from the general government which accounts for the fact that, in many countries, social security budgets are partly financed through tax revenues. We assume that the social security budget is balanced in each period. Therefore, the budget constraint is given by

$$\begin{aligned} & \sum_{i,a} \left(\tilde{P}_i^a + \tilde{PDB}_i^a\right) - ex^{Pens,Corr} + \sum_{i,a} B_i^a + p^{cg} C^H \\ = & \sum_{i,a} \left(t_i^{sscW,a} + t_i^{sscF,a}\right) W_i^a + \sum_{i,a} t_i^{sscP,a} \left(\tilde{P}_i^a + P\tilde{D}B_i^a\right) + Z^{ssc} + Tr, \end{aligned} \tag{90}$$

so that aggregate expenditures for old age pensions $\tilde{P}_i^a$, disability pensions $P\tilde{D}B_i^a$ (corrected by pension benefits paid by foreign governments $ex^{Pens,Corr}$), unemployment benefits B_i^a and public health expenditures C^H are financed by contributions of employers, employees (wage-related and flat) and retirees, and a transfer from the general budget. W_i^a is a properly defined assessment base for social security contributions.

The primary balance of the *general public budget* is given by

$$PB = rev - ex, \tag{91}$$

where the revenues of the public budget are given by:

$$\begin{aligned} rev = & \sum_{i,a} t_i^{W,a} \left(1 - x_i^{W,a} t_i^{sscW,a}\right) W_i^a + \\ & + \sum_{i,a} t_i^{P,a} \left(1 - x_i^{P,a} t_i^{sscP,a}\right) \tilde{P}_i^a + \\ & + T^F + \sum_{i,a} t_i^{c,a} \frac{pc_i^a}{1 + t_i^{c,a}} C_i^a + T^{CG} + Fir. \end{aligned} \tag{92}$$

Revenues of the general budget consist of income taxes, taxes on firms, consumption taxes, capital gains taxes, and firing taxes. Expenditures are

$$ex = p^{cg} C^G + Tr + Z, \tag{93}$$

where C^G is public consumption, Tr are transfers to the social security budgets, and Z are lump sum transfers and social assistance paid to households as well as subsidies paid to firms. As can be seen from the utility functions, we assume that public consumption does not result in utility gains of the households.

The government debt accumulation equation is

$$G \cdot DG_{t+1} = R_{t+1} \left(DG_t - PB_t\right). \tag{94}$$

[28] We do not distinguish different levels of governments like federal, states or local.

In the standard case, we impose the condition that $DG_{t+1} = DG_t$, $\forall\ t$, i.e. that the government debt is constant over time as a share of initial GDP. [29] Thus, if the government is indebted, it has to run a primary surplus to keep the debt ratio constant if $R > G$. We also assume that the social security system has to be balanced in each period.

There may be different rules to balance the different budgets. As for the social security budget, one may either adjust the social security contribution rates, cut benefits or increase transfer payments from the general budget. The general budget may be balanced by an increase of taxes (e.g. VAT, income tax, lump sum transfers) or by a reduction of government consumption. These rules can be simply set by changing parameters in the model.

[29] However, alternative rules for the path of government debt can be implemented rather easily with some programming effort.

8 Equilibrium

Gross value added in nominal terms[30] can be derived by the expenditure approach by adding final demand for private and public consumption, investment and the trade balance or the production approach by subtracting the different costs of the firm from produced output as defined in equation (76)[31], i.e.

$$GVA = \sum_j p_j \bar{Y}_j \tag{95}$$

The trade balance is defined as GVA minus domestic demand:

$$TB = GVA - p^{inv} I - pc \cdot C - p^{cg} \left(C^G + C^H \right). \tag{96}$$

The stock of foreign assets DF evolves with the trade balance, i.e.

$$\begin{aligned} G \cdot DF_{t+1} &= R_{t+1} \left[DF_t + TB_t - ex_t^{Pens,Corr} \right] + \\ &+ A^{adj,a} \frac{netmig_{t+1}^a}{N_{t+1}^a - netmig_{t+1}^a} R_{t+1}^{\tau} \left[\omega^a Sav_t^a + \left(1 - \omega^{a-1} \right) Sav_t^{a-1} \right]. \end{aligned} \tag{97}$$

Thus, migration adds two terms to a standard dynamic formula for foreign assets. First, the 'pension transfer' $ex_t^{Pens,Corr}$ paid by foreign governments. Second, migrants transfer their private assets, which adds to the stock of foreign assets of the economy. Finally, the asset market clears in a standard fashion as net financial assets of households are comprised of holdings of public debt, firm values and foreign assets:

$$A = DG + VF + DF. \tag{98}$$

Walras' law implies that the sum of the values of excess demands across all modelled markets must equal zero. This condition, which is shown in the Appendix, holds independent of whether or not the economy is in general equilibrium.

9 Welfare Analysis

Our welfare computations are based on equivalent variations which give the change in income or wealth that is equivalent to the change in utility assuming constant prices. Equations (17) and (45) derive indirect life-time utility $V_{\alpha,t}^a$ of an agent in age group a with history α. We compute separately the welfare per capita of a newborn (they are not yet heterogeneous in their history) and the welfare of an average agent in age group a. We express all values per capita, and use $\sum_{\alpha \in \mathcal{N}_t^a} A_{\alpha,t}^a N_{\alpha,t}^a = A_t^a \equiv \bar{A}_t^a N_t^a$ to get average assets $\bar{A}_t^a$ per capita. Aggregation of value functions per capita in group a gives

$$V_t^a = P_t^a \cdot \mathcal{W}_t^a, \quad P_t^a \equiv \left(\Delta_t^a \right)^{1/(\sigma-1)}, \quad \mathcal{W}_t^a \equiv \bar{A}_t^a + H_t^a + S_t^a + T_t^a. \tag{99}$$

[30]The model neglects inflationary price trends. For this reason nominal terms is not the correct term. However, price changes due to changes of supply and demand are reflected. In a steady state, growth rates of real and nominal GDP coincide.

[31]GDP follows from GVA by adding taxes on products and subtracting subsidies on products.

Aggregate welfare of age group a is simply $V_t^a N_t^a$. Inverting indirect utility yields an intertemporal expenditure function which gives the level of life-time wealth per capita necessary to obtain utility V_t^a at intertemporal prices P_t^a,

$$\varrho\left(P_t^a, V_t^a\right) = V_t^a / P_t^a. \tag{100}$$

When considering the welfare impact of a policy shock, we denote equilibrium values *before* a policy shock by an upper index of zero and delete the index if we refer to values after the shock, $P_t^{0,a}$ and P_t^a, for example. Taking initial prices as a reference, the equivalent variation measure EV gives the wealth equivalent change in welfare per capita. It is defined (for each generation or population group separately) as

$$EV_t^a = \varrho\left(P_t^{0,a}, V_t^a\right) - \mathcal{W}_t^{0,a} = V_t^a / P_t^{0,a} - \mathcal{W}_t^{0,a}. \tag{101}$$

9.1 Aggregate Welfare Measure

Suppose the policy change becomes effective in period $t = 1$. To characterize intergenerational redistribution, we distinguish between present and future generations. We report the aggregate welfare impact for each age group living in period $t = 1$. Existing present generations were born in periods $t \leq 1$. The equivalent variation may be expressed as a percentage of life-time wealth $\mathcal{W}^{0,a}$.

We separately report the results for all new, future generations entering the economy in periods $t > 1$. Note that the newborns $N_{t,t}^1$ are always a fraction of the first age group with total size N_t^1. New agents start life without any assets and social security wealth.

The aggregate equivalent welfare change of new generations born in period 2 and afterwards is

$$EV_1^N = \sum_{t=2}^{T-1} EV_{t,t}^1 \cdot N_{t,t}^1 \prod_{u=2}^{t} \frac{G}{R_u} + \frac{R}{r-g} EV_{T,T}^1 \cdot N_{T,T}^1 \prod_{u=2}^{T} \frac{G}{R_u}, \tag{102}$$

where $EV_{t,t}^1$ is the equivalent variation of an individual born in t. The first term adds up the variations of new generations born before period T. The second term collects all those born in period T or afterwards where T is the length of the transition period. In period T, the economy has already approached a steady state, giving constant values thereafter. The last multiplicative term discounts the welfare gains of all new generations born in periods T or later to period 1.

Adding up welfare changes of new and old generations, we can derive an aggregate measure,

$$EV_1 = EV_1^N + EV_1^O, \quad EV_1^O = \sum\nolimits_a EV_1^a \cdot N_1^a. \tag{103}$$

The aggregate welfare change is equivalent to an amount of wealth EV_1, as measured at the beginning of period 1. To express welfare gains relative to GDP, we convert this wealth measure into permanent income, i.e. we compute a permanent annuity with the same present value, taking the initial interest rate as the discount rate,

$$EV_1 = ev + ev\frac{G}{R} + ev\frac{G}{R^2} + \ldots = ev\frac{R}{r-g} \quad \Rightarrow \quad ev = \frac{r-g}{R} EV_1. \tag{104}$$

The aggregate welfare change in percent of GDP is then $100 \times ev/GDP^0$.

10 Functional Forms

10.1 Functional Forms Concerning Firms

The **production function** used in the model is a nested CES-production function Y. The different skill-groups and capital input are the nests. Capital and aggregate effective[32] labour input are the inputs for the production function.

$$Y = F^{Y}(uK, L_1^D, L_2^D, L_3^D) = A \cdot z_1,$$

where

$$\begin{aligned} z_1 &= \left[a_1 \left(L_1^D\right)^{\pi_1} + (1-a_1)\, z_2^{\pi_1}\right]^{1/\pi_1}; \\ z_2 &= \left[a_2 \left(L_2^D\right)^{\pi_2} + (1-a_2)\, z_3^{\pi_2}\right]^{1/\pi_2}; \\ z_3 &= \left[a_3 \left(L_3^D\right)^{\pi_3} + (1-a_3)\, (uK)^{\pi_3}\right]^{1/\pi_3}. \end{aligned}$$

Given the functional form of the production function the marginal products of labour and capital are given by:

$$\begin{aligned} F_{L,1}^{Y} &= A \cdot a_1 \left(L_1^D\right)^{\pi_1 - 1} z_1^{1-\pi_1}; \qquad (105) \\ F_{L,2}^{Y} &= A\,(1-a_1)\, a_2 \left(L_2^D\right)^{\pi_2 - 1} z_1^{1-\pi_1} z_2^{\pi_1-\pi_2}; \\ F_{L,3}^{Y} &= A\,(1-a_1)\,(1-a_2)\, a_3 \left(L_3^D\right)^{\pi_3 - 1} z_1^{1-\pi_1} z_2^{\pi_1-\pi_2} z_3^{\pi_2-\pi_3}; \\ F_{K}^{Y} &= A\,(1-a_1)\,(1-a_2)\,(1-a_3)\, u^{\pi_3} K^{\pi_3-1} z_1^{1-\pi_1} z_2^{\pi_1-\pi_2} z_3^{\pi_2-\pi_3}; \\ F_{u}^{Y} &= A\,(1-a_1)\,(1-a_2)\,(1-a_3)\, u^{\pi_3-1} K^{\pi_3} z_1^{1-\pi_1} z_2^{\pi_1-\pi_2} z_3^{\pi_2-\pi_3}. \end{aligned}$$

Final goods firms decide about the utilization of the rented capital stock, i.e. how extensive the capital stock is used in the production. Higher intensity leads to additional output with costs of additional depreciation. We assume convex deprecation costs, such that depreciation rises more than proportionally. The functional form for firm j is given by:

$$\delta(u_j) = a_\delta + b_\delta u_j^{c_\delta}, \qquad (106)$$

where u_j stands for the utilization of capital (with $u = 1$ for average utilization of the capital stock). The variable a_δ reflects depreciation if the capital stock is not used at all, i.e. $u = 0$, and implies decompositional or technological ageing effects. The sum of a_δ and b_δ is the depreciation rate during times of normal use of the capital stock.

Whenever firms want to adjust their capital stock as a result of a reform scenario they have to bear **installation costs**. The installation cost function J is given by:

$$J = \frac{\psi}{2}\left(j - \delta(u) - g\right)^2 K, \qquad (107)$$

[32] Taking into account productivity units of the different age groups.

where ψ is a scaling factor, j is the investment-capital ratio $j = \frac{I}{K}$, δ the depreciation rate and g the steady state growth rate. The parameter ψ allows to coincide capital adjustment durations with empirical estimations. In the steady state, the investment-capital ratio j is equal to the sum of depreciation and growth, $j = \delta(u) + g$, so that the installation costs are zero.

The functional form of the **vacancy costs** is given by:

$$\begin{aligned} \kappa(v) &= v_0 + \frac{\kappa_0 \left(\frac{v}{L^X}\right)^{1+1/\varepsilon} L^X}{1+1/\varepsilon}, \\ \kappa'(v) &= \kappa_0 \left(v/L^X\right)^{1/\varepsilon}. \end{aligned} \tag{108}$$

where $L^X = \sum_a \delta^a \bar{\delta}^a (1-\varepsilon^a) N^a$ is the number of persons searching for a job and $\varepsilon > 0$. The cost function is a convex function of the vacancies per person searching for a job, implying increasing costs.

We follow Ejarque and Portugal (2007) and introduce the following functional form of **labour adjustment costs**:

$$Ladj_i\left(N^W_{t,i}, N^W_{t-1,i}\right) = \frac{H_i}{2} \frac{\left(N^W_{t,i} - N^W_{t-1,i}\right)^2}{N^W_{t-1,i}},$$

where $N^W_{t,i} = \sum_a p^a_{man,i} L^{H,a}_i$ is the number of workers in a certain skill group,

$$Ladj_{N,i} = H_i \frac{\left(N^W_{t,i} - N^W_{t-1,i}\right)}{N^W_{t-1,i}},$$

is the derivative w.r.t. the first argument.

Managerial costs reflect the effort costs of the firm arising from keeping their workers. An increase of the firing rate, $(1 - p_{man})$, decreases these costs. Given that $v_F > 0$, the cost function is convex in p_{man}.

$$\begin{aligned} \varphi^F(p_{man}) &= (p_1)^{-1/v_F} \frac{p_{man}^{1+1/v_F}}{1+1/v_F} - p_0, \\ \varphi^{F\prime}(p_{man}) &= \left(\frac{p_{man}}{p_1}\right)^{1/v_F}. \end{aligned}$$

The functional form of costs for **firm-sponsored training** is assumed to be the same:

$$\begin{aligned} \varphi^{FS}\left(e^F\right) &= fs_1 \frac{\left(e^F\right)^{1+1/v_{FS}}}{1+1/v_{FS}} - fs_0, \\ \varphi^{FS\prime}\left(e^F\right) &= fs_1 \left(e^F\right)^{1/v_{FS}}. \end{aligned}$$

We assume a simple form for the translation of firm-sponsored training into **firm-specific productivity**:

$$\theta^F\left(e^F\right) = A^F \left(e^F\right)^{\alpha_{FS}} + 1,$$

so that the factor $\theta^F = 1$ if firms provide no training.

10.2 Functional Forms Concerning Households

Remember that, as described above, we normalize the different effort costs of the household by firm-specific productivity θ^F for notational reasons. The disutility of **labour effort**, i.e. disutility with respect to the number of hours worked or spent in training, is given by:

$$\varphi^{L}\left(L\right)=\left[\left(l_{0}\right)^{-1/v_{l}}\frac{\left(L_{i}+c\right)^{1+1/v_{l}}}{1+1/v_{l}}\right]\frac{1}{\theta^{F}}.$$

The functional form is equivalent to the function for managerial costs, also being convex in L_i^a, given that $v_l > 0$. The constant c is added so that we can implement some policy scenarios more easily. The elasticity v_l is skill-dependent in the model. Unemployed persons search for a new job. The time spent searching leads to a **disutility of searching**. We apply a similar functional form as for the disutility of labour effort (meaning a convex function that is increasing in the search effort).

$$\varphi^{S}(s)=\left[\left(s_{0}\right)^{-1/v_{u}}\frac{s^{1+1/v_{u}}-\left(s_{calib}\right)^{1+1/v_{u}}}{1+1/v_{u}}\right]\frac{1}{\theta^{F}},$$

The increase of participation of a household not only lowers the amount of goods produced at home in a linear way, but also leads to additional non-linear **disutility of participating**. The disutility of participating is normalized to zero in the initial steady state and is given by:

$$\varphi^{P}\left(\delta\right)=\frac{\delta_{0}}{\theta^{F}}v^{r}\left[\exp\left(\frac{\delta}{v^{r}}\right)-\exp\left(\frac{\delta_{calib}}{v^{r}}\right)\right].$$

Being employed leads to an increase of human capital in the model as long as persons allocate time to life-long learning activities. The additional human capital formation per efficiency unit $\theta_{\alpha,i}^{a}$ is derived by the following **human capital production function**:

$$F\left(E\right) = F_{0}E^{\alpha} \tag{109a}$$

$$F_{E} = \alpha F_{0}E^{\alpha-1}, \tag{109b}$$

$$F_{e} = \delta\bar{\delta}\left(1-u\right)F_{E},$$

$$E = \delta\bar{\delta}\left(1-u\right)e.$$

where F_0 represents a factor productivity and e the time spent in this activity.

10.3 Functional Forms Concerning Matching in the Labour Market

The matching function for each age and skill group is given by the standard linear homogeneous function M_i^a:

$$M_{i}^{a}=m_{0,i}^{a}\left(S_{i}^{a}\right)^{\sigma}\left(V_{i}^{a}\right)^{1-\sigma},$$

where $S_i^a = (1-\varepsilon)\, s_i^a \delta_i^a \bar{\delta}_i^a N_i^a$ is the aggregate search intensity of age group a and skill group i and $V_i^a = \sum_j v_{j,i}^a$ the sum of vacancies across the firms in the economy. Given this functional form, the probability of filling a vacancy q_i^a and finding a job f_i^a are given by:

$$q_{i}^{a} = \frac{M_{i}^{a}}{V_{i}^{a}}=m_{0,i}^{a}\left(\Theta_{i}^{M,a}\right)^{-\sigma};$$

$$f_{i}^{a} = \frac{M_{i}^{a}}{S_{i}^{a}}=\Theta_{i}^{M,a}q_{i}^{a}=m_{0,i}^{a}\left(\Theta_{i}^{M,a}\right)^{1-\sigma},$$

where $\Theta_i^{M,a}$ defines the labour market tightness for the corresponding age and skill group:

$$\Theta_{i}^{M,a}=\frac{V_{i}^{a}}{S_{i}^{a}}.$$

11 Calibration

This section describes the calibration of the model. It gives details on the data collection of macroeconomic variables, household characteristics, the production side and the public sector. Our model distinguishes between skill- and age-groups, hence we need a considerable amount of disaggregated household data. Since most of these data are not publicly available from official institutions, we have to get or calculate them from individual data sets, i.e. the LFS and the EU-SILC. This section also includes a review of the empirical literature on behavioural parameters of firms and households relevant for the model.

11.1 System of National Accounts

11.1.1 System of National Accounts and the Capital Stock, Exogenous Growth Path and World Interest Rate

In this section, we discuss the different macro(economic) data which are used to calibrate the model. They are in general based on officially available harmonised data and can be updated easily.

The system of national accounts (SNA) is the basis to calibrate major aggregates, like private and public consumption, investment, the capital and wage income shares, etc. However, we do not use these values one to one in the model. We describe below how we transformed the data for our use. As investment and capital stock are closely related, we also describe here the adjustments and data source for the capital stock.

The capital stock in the economy is taken either from the SNA or AMECO-database. Depreciation rates are derived in the calibration procedure of the model. The model does not distinguish between employees and self-employed persons, and labour compensation contains labour income of both types. Therefore, we distribute a share of the ‘operating surplus and mixed income’ to the compensation of employees to calculate the labour income share. We assume that the distribution of self-employed persons according to age and skills is similar to that of the employees, that they earn the same labour income per hour, but work a different number of hours.

A further adjustment refers to gross value added and GDP in the model. Gross value added corresponds to GDP minus taxes on products (d21 in the notation of SNA) plus subsidies on products (d31). Production taxes (and subsidies) on intermediary production (d29 and d39) are interpreted as taxes on capital or as additional taxes on labour (e.g. local tax in Austria). We deviate from the SNA by using taxes on consumption from the OECD Revenue Statistics instead of d21 to derive gross value added to be consistent within the model as these tax revenues are used to reflect the consumption tax revenues.

11.1.2 Revenues of the Public Sector

Public revenues from taxes and social security contributions are derived by using detailed data from OECD’s database on Revenue Statistics (see e.g. OECD (2016)). We group the detailed items according to their economic function in five categories: Income, Capital Gains, Corporates, Social Security Contributions and Consumption.

These 'Model Revenues' in the five different categories are used to calibrate PuMA's tax rates. Some tax rates (tax on consumption and capital gains tax) are calculated directly by relating revenues to the assessment base (i.e. private consumption or capital gains). For instance, the consumption tax rate is equal to revenues from taxes on consumption divided by the assessment base for consumption tax, consisting of total private consumption and a share of public consumption (intermediate consumption and consumption of fixed capital).

Income tax rates and social security contribution rates of employers and employees according to education and age are derived via the method described in Section 11.2.3. Simply speaking, we derive age- and skill specific tax and social security contribution rates using OECD'S Tax-Benefit model and income data from the EU-SILC. These rates are subsequently adjusted for all groups so that we get appropriate revenues (only minor adjustments of the initially calculated rates are necessary).

For corporate taxation, a different method is applied and we use the calculations of the ZEW (2014) effective corporate tax rates which are based on the method by Devereux and Griffith (2003). In a second step, we calibrate the necessary deductions of the corporate tax base so that revenues in PuMA fit to revenues based on OECD data.

11.2 Household Characteristics

11.2.1 Demography

PuMA includes a detailed breakdown of the population with respect to age and educational attainment, so that we can analyse both age- and skill-dependent impacts of policy reforms, as also indicated in the illustrative reform scenario, see Annex A. PuMA distinguishes three different groups of educational attainment. The low-skilled group comprises individuals with pre-primary, primary and lower secondary education (ISCED 0-2 according to the ISCED classification), individuals with completed tertiary education (ISCED 5+) are high-skilled and medium-skilled individuals have an upper secondary (and post-secondary non-tertiary) level of education (ISCED 3-4). The distribution of the 25 to 64 years old population according to the highest level of education attainment is based on Eurostat data.

The model is calibrated to an initial steady state that also assumes a stationary demographic structure which implies that the demographic structure of the population in the model deviates from the actual demographic structure. Our approach is that we take current mortality rates for each one-year-cohort from Eurostat and derive average mortality rates for our age groups. Thereby, we usually overestimate the group of older individuals compared to the real data. However, given that we adjust the flat pension in order to derive actual current pension expenditures (see Section 11.7.1), this is probably the best way to deal with this issue.

11.2.2 Labour Market Information from LFS and EU-SILC

The model exhibits a high degree of heterogeneity on the household side because we distinguish households according to both age and skills. This leads to a considerable demand for data on the labour market status, behaviour, and also institutional details. In general, such specific requirements are not provided by standardized databases. Therefore, it is necessary to use individual data to derive the necessary inputs for the model, namely for each age- and skill-group. In general, data is based on two sets containing individual level data for the EU-countries: the Labour Force Survey (LFS) and the Community Statistics on Income and Living Conditions (EU-SILC). We first describe the general structure of the data sources that we use to derive the parameters required for the calibration of PuMA. Then, we explain the data corrections we applied to make them usable for our model. As often as possible, we cross-checked these variables with Eurostat data.

Labour Force Survey (LFS) The *Labour Force Survey* is designed to obtain information from households about the labour market and related issues by means of personal and telephone interviews. The LFS not only collects information on persons in the labour force, but also on other persons present in the households. The LFS is a continuous survey providing quarterly and annual results. The sampling rates in the different countries vary between 0.2 percent and 2.1 percent of the population. Persons in collective households and institutions are excluded. The strength of the LFS lies in its comparability across the Member States, because of recording the same set of characteristics, the use of common classifications and definitions, and central processing of data. The data set contains relevant information of the data necessary to fit the model. The survey includes, for example:

- A derived variable age, aggregated into groups with a spread of five years (year and date of birth are not available for anonymous datasets)
- The highest level of education attained according to the ISCED definitions to assign individuals to skill groups, as well as the current status of education (see Section 11.2.2 for further discussion) to correct for some of the younger age-skill groups
- Sex, marital status, region of birth
- The distinction between employed, unemployed and inactive
- The distinction between employee and self-employed
- The number of hours worked
- The year and month in which the person started working for the employer
- The information whether someone is currently searching for a job
- Participation in education and training within the last four weeks, the total number of hours spent on all these learning activities within the last four weeks, and whether education and training are relevant for a job or not

- Reasons for leaving a certain employment, e.g. because of dismissal, limited duration of the job, children, illness or disability

Furthermore, the weighting factor allows for aggregation, whereby consistency checks with other statistics are done if possible. Due to the extensiveness of the data, the LFS is the main database for the labour market variables in the model other than income (the calibration of income variables is mainly based on the EU-SILC).

Community Statistics on Income and Living Conditions (EU-SILC) The EU-SILC is an instrument aimed at collecting timely and comparable cross-sectional and longitudinal multidimensional micro data on income, poverty and social exclusion (European Commission (2008)). EU-SILC was launched in 2004 in 13 Member States and reached almost full scale extension in 2005 by providing data for 25 Member States. Mostly, data that we take from the SILC are about the income of households and individual household members. The European cross sectional sample size is about 135,000[33] households. The number of households in the countries should ensure a minimum precision for each country. The annual data have a large overlap for the longitudinal analysis. In simple terms, once a household is selected, it is also part of the sample in the three consecutive samples. Therefore, there is an overlap of households of 75 percent between year T and year $T + 1$. There are differences in the sampling of the households conducted by each country. More details on this issue may be found in the *EU-SILC User Database Description*.

EU-SILC contains a lot of information about households, but also on individual levels. It provides data about important aspects of households, such as number of children, total household income, gross income components at the household level, and social criteria. On the individual level EU-SILC contains for example:

- The year and quarter of birth
- The ISCED level currently attained
- Number of hours usually worked per week in the main job, but also in second and third jobs
- The number of months spent in full-time and part-time work, in unemployment, in retirement, studying and inactivity
- Status in employment
- Yearly earnings for employees
- The employee cash or near-cash income as well as non-cash employee income, cash benefits and losses from self-employment
- Other forms of income (unemployment benefits, old-age, survivor, sickness, and disability benefits)

[33]See Eurostat, Statistics Explained

EU-SILC provides us mainly with data about the labour status of persons and different types of income. In particular data on unemployment income and other transfers are important, as most of these are completely missing in the LFS.

Pooling, Corrections and Problems

Pooling of the Data We pool the available data of several years of the LFS as well as of the EU-SILC. In general two reasons lead to this decision:

- Sample sizes broken down for the different age- and skill-cells are sometimes very small. Pooling of data is a common way to increase reliability.
- We want to smooth the impact of the business cycle on the surveyed data. As the business cycle influences markably some of the variables, pooling decreases cyclical effects.

Adjustments Relating to the LFS We describe here the data adjustments we apply. First, our model requires a certain kind of input for the low-skilled aged 15-19 and 20-24 years, as well as for the medium-skilled persons aged 20-24. In the model they are 'born' with the skill-level assigned to them. A 20 year old student does not work. To account for this we remove persons currently undergoing educational activities (LFS variable: $educstat = 1$) from the age-skill groups mentioned above when we derive labour market relevant variables from the official statistics. When calculating the unemployment rate for 15-19 year old low-skilled workers, we removed all persons still undergoing education. This leads to an unemployment rate for this age-skill cell that is higher than we would expect from other statistical sources. An obvious explanation after removing people still undergoing formal education, we are left with a very high share of persons with only a basic education not even trying to get further education. This causes a strong upward bias of the unemployment rate for our calculations. A second important reason is, that a share of low-skilled persons is searching for an apprenticeship training position. In the data, these individuals are low-skilled and unemployed. However, they will get an apprenticeship degree (and thus become medium-skilled) in the future. We have solved this problem by replacing the unemployment rate for the low-skilled aged 15-19 years with the one of the low-skilled aged 20-24, since this value seems to be the closest approximation.

Wage Regressions with EU-SILC Data We estimate the returns to education, experience and other factors from the available data sources. The theoretical framework for such estimations was developed by Mincer (1974). The empirical approximation of the human capital theoretical framework is the familiar functional form of the earnings equation:

$$logw_i = X_i\beta + rs_i + \delta x_i + \gamma x_i^2 + u_i, \tag{110}$$

where w_i is an earnings measure for an individual i such as earnings per hour/week, s_i represents a measure of her schooling, x_i is an experience measure (in our case age minus average years of schooling),

X_i is a set of other variables assumed to affect earnings, and u_i is a disturbance term representing other factors which are not measured explicitly. Experience is also included as a quadratic term in order to describe the concavity of the earnings profile. In the above equation, r is the private return to schooling.

The literature regarding this topic is very extensive and the approaches to it are various. Specifically, OLS estimates seem to be biased downwards (Griliches, 1977, Angrist and Krueger, 1999) by measurement error, even though recent evidence (Card, 2001) only allocates at most ten percent gap to this fact. By contrast, since an unobserved ability determining education could have a large effect on the education attained, the schooling coefficient might be biased upwards (Griliches, 1977). Since a direct indication of this ability such as IQ test scores or information about the wages of identical twins (see Ashenfelter and Krueger, 1994, and Miller et al., 1995) is not available in most cases (and specifically also in our case), some authors use an instrumental variable (IV) approach. These instruments should correlate with schooling but not with the error term of the regression.

Usually, IV estimations tend to be higher than OLS estimates. This is a rather surprising result, since, as already mentioned above, the OLS estimates are expected to be biased upwards because of some unobserved ability bias. Various explanations were given for this result, such as that the IV estimates have an upwards bias (Ashenfelter et al., 1999), or that the downward bias in OLS attributed to the measurement errors dominates the upwards bias (Angrist and Krueger, 1991). The latter explanation seems unlikely, since it was shown by Card (2001) that this downward bias can account for mostly 10 percent of the gap between OLS and IV estimates. One explanation (Card, 1999, 2001) for this apparent upward bias of IV estimates is that the returns of education are heterogeneous across individuals and that the IV estimates tend to recover the returns to education of the population group most affected by the intervention, hence they would be a better approximation for the returns to education of the affected group rather than for the whole population.[34] Because of these various problems and the complexity of the estimation process, we decided to estimate an OLS regression.

We use a 'reduced form' approach in our specification for the earnings function. There are plenty of other variables typically known to influence income, like the type of job, the industry and firm a worker is employed in, etc. While these variables can purge the schooling coefficient from influences which are not directly caused by education but by a subsequent choice of industry or job type, this is not what we intend to do. Our schooling coefficient is supposed to cover all direct and indirect consequences of a schooling decision. Therefore, we do not include any variables - other than pre-determined ones - in our earnings equation.

When estimating wage regressions, we remove the self-employed from the dataset since their income includes not only labour income but also capital income. Additionally, we remove individuals with missing reported values from the data set in the estimation of the wage regressions. This is the case, for instance, if we can not calculate the average monthly income because a certain person had not reported the months worked during the income reference period.

One problem when running the estimation of the wage profile based on survey data is the following:

[34] See Garcia-Mainar and Montuenga-Gomez (2008) for an extensive literature survey.

the EU-SILC contains information on labour income from the *income reference period*, which is the year *previous* to the one in which the survey takes place, while the number of hours worked per week refers to a period just before the survey takes place. Since our wage regressions are based on hourly wages which we derive from the two variables mentioned above, we calculate hourly wages with hours *currently* worked and compare it to income generated in the income reference period. This might cause distortions of hourly wages.[35]

11.2.3 Tax and Social Security Contribution Rates

The model's detailed breakdown of private households according to age and educational groups allows for a different tax and social security contribution rates of different groups to consider, for example, progressive income tax systems, maximum thresholds for social security contributions or earned income tax credits. The 'drawback' of this detailed representation is the considerable calibration effort. Whereas tax rates can be calibrated rather easily by using aggregate revenue data in 'standard' models with only one representative household, the calibration of the PuMA relies on a sophisticated (and rather time-consuming) application of OECD's Tax-Benefit model (using institutional details based on the year 2014) on EU-SILC data. However, once the calculation method is completed, tax and social security reforms can be replicated rather easily and in profound detail.

We don't want to set identical tax rates for all individuals and we are not aware of any dataset that could give us tax rates in sufficient detail. We apply our own method by proceeding in the following way. The OECD provides Tax-Benefit models [36] for the countries included in PuMA. The Tax-Benefit model is used to calculate personal income tax and social security payments for different types of households and amounts of income. When using the OECD model directly, by changing parameters and running the Stata code, we can receive a much more detailed breakdown of household types and income than those reported OECD's Taxing Wages 2017. In particular, we run simulations and calculate tax rates for Singles, One- and More-Earner-Households, for a different number of children and a very detailed breakdown of income.[37] Subsequently, we apply the obtained tax rates to our EU-SILC dataset. We classify the households in the EU-SILC according to the parameters family type, number of children and income. In this way, we can calculate tax and (employee's and employer's) social security rates for our breakdown of age and educational groups.

11.2.4 Consumption Profile and Elasticity of Intertemporal Substitution

The consumption and savings decisions of individuals, in which individuals decide how much to consume or save, determine an optimal marginal propensity to consume ('mpc'$= 1/\Delta^a$) out of expected total lifetime wealth (which consists of financial wealth and the present value of future labour and pension income and transfers). As described in equation (45), this 'mpc' is age-dependent and determined by different

[35]For further information about problems with calculating hourly wages, see Tijdens et al. (2005), p. 10.
[36]See the discussion in OECD's Benefits and Wages (2007) publication on the strengths and limitations of the model.
[37]We break down the income in percentiles, ranging from two to four-hundred percent of the income of the average worker.

parameters such as preference parameters (e.g. the subjective discount factor and the intertemporal elasticity of substitution), policy parameters such as the consumption tax rate and future mortality rates (i.e. remaining life expectancy). Thus, combined with the stream of income and transfers, the 'mpc' determines an optimal intertemporal consumption profile of individuals in economic models. However, this consumption profile can deviate from the profile actually observed in real data. To avoid this deviation, PuMA incorporates inter-vivo transfers between households. We calibrate these transfers such that the consumption profile observed in reality results from optimal household behaviour and income streams in the model. Given calibrated values for income and transfers and the consumption profile, the asset profile is endogenously determined as a result of the intertemporal budget constraint of private households. Data on private consumption expenditures per adult equivalent for different age groups are taken from Eurostat. The dataset contains few data on very young and very old households. Therefore, we estimate quadratic consumption profiles, which usually results in the expected hump-shaped consumption profile.

Empirical estimates for the elasticity of intertemporal substitution (EIS, σ) range widely between 0 to 2. Engelhardt and Kumar (2008) provide a short summary of estimated values for the EIS. Some of the estimates of the EIS are close to zero, like in Hall (1988). Blundell et al. (1993) find a value of 0.5 for the EIS, Attanasio and Weber (1995) estimate the value in a range from 0.6 to 0.7, Vissing-Jorgensen (2002) from 0.3 to 1, and Ziliak and Kniesner (2005) from 0.7 to 1. Higher estimates are provided by Mulligan (2002), Gruber (2006), and Vissing-Jorgensen and Attanasio (2003) from 1 to 2. The own estimate of Engelhardt and Kumar is 0.74, with a 95 percent confidence interval that ranges from 0.37 to 1.21. Gruber (2006) states that micro-data estimates mostly provide an EIS from 0.1 to 1. As an illustration, we also provide some values for the EIS that are used in economic models. Altig et al. (2001) apply an EIS of 0.25, Kumhof and Laxton (2007) 0.25, Botman and Kumar (2006) 0.33, Jaag et al. (2007) 0.35, Trostel (1993) 0.8, Bovenberg et al. (1998) 0.85, and Perroni (1995), Mankiw and Weinzierl (2004), and Trabandt and Uhlig (2006) use an EIS of 1. We use a value of 0.4 in our model. However, sensitivity analysis suggests that the value of the EIS is not an important determinant of the effects of labour market policy reforms in PuMA.

11.2.5 Elasticities on the Household Side

Labour Supply Elasticities There is a large number of studies on labour supply elasticities and we mention only a few here. In their simulation study, Immervoll et al. (2007) concentrate on substitution effects, because this considerably simplifies the analysis. They say that empirical evidence points to the fact that there may be significant income effects for married women and single mothers, but no income effects for males. They argue that, as they are considering balanced budget reforms, this 'mistake' is not too big. Blundell and MaCurdy (1999) survey existing approaches to modelling labour supply and summarize a selection of empirical findings. In their Tables 1 (for men) and 2 (for married women), the substitution elasticity exceeds the income elasticity for the vast majority of papers. For tractability of the model, we follow Immervoll et al. in assuming utility functions that neglect the income effect.

Evers et al. (2005) perform a meta-analysis on uncompensated labour supply elasticities by using 239 elasticities from 32 different empirical studies. They focus on uncompensated elasticities because they cannot obtain separate values from many primary studies for the compensated elasticity and the income elasticity. Regression results of Evers et al. indicate that, even after controlling for the participation decision, women still have higher hours-of-work labour supply elasticities. Thus, they somehow contradict Immervoll et al., who claim that, once labour supply is estimated conditional on labour force participation, it turns out that female hours-of-work elasticities are close to that of males. Evers' meta-regressions give 'total' elasticities between 0.07 and 0.16 for Dutch males and 0.48 and 0.52 for Dutch females. If they control for participation, so that they get point estimates for the intensive margin, they get elasticities of 0.44 resp. 0.03 for Dutch women and men.

There is wide consensus that extensive labour supply responses may be much stronger than intensive responses (Heckman, 1993)) and that participation elasticities tend to be high at the lower end of the earnings distribution. Immervoll et al. calibrate their model with an average participation elasticity of 0.2 (ranging from 0.4 for the lowest two income deciles to 0 for the highest two). As regarding the hours-of-work elasticity, they assume it to be constant across deciles (similar to Diamond (1998) and Saez (2001)) and equal to 0.1. Meghir and Phillips (2008) provide an overview of literature on labour supply with a focus on presenting the empirical consensus. They argue that male hours of work are very irresponsive, but male participation can be responsive, especially for low- and medium-skilled. They claim that hours of work and participation for women with young children and lone mothers are very sensitive.

Finally, Evers et al. notice that hardly any evidence can be found supporting the hypothesis that elasticities differ between countries. This result is confirmed by Immervoll et al., who claim that evidence from both estimation and direct policy analysis suggests magnitudes across all European countries to be similar to those obtained in the literature on Anglo-Saxon countries. Accordingly, we set similar labour supply elasticities for all countries in PuMA. We set the intensive labour supply elasticity equal to 0.1 for low-skilled and slightly lower for medium- and high-skilled individuals. For the participation semi-elasticity w.r.t. income, we set slightly lower values than Immervoll et al. (2007), i.e. 0.2, 0.15 and 0.08 for low-, medium- and high-skilled individuals, respectively.

Retirement Elasticities The retirement age seems to be an important behavioural margin of households that is neglected in most simulation studies. In their seminal study, Gruber and Wise (2002) present international research on the relationship between social security provisions and the retirement decision. Based on three different incentive measures of pension systems, namely accrual, option value and peak value, they show that there is broad consensus in the different country studies that these incentives have a strong effect on retirement decisions. However, although the authors present probit estimates and 'standardized' simulations for all countries, it is difficult to entangle a parameter that would fit into our model from these estimates. An exception are Börsch-Supan et al. (2004) who find an increase of effective retirement age of 8 months in response to the introduction of a penalty factor of 3.6 percent per year of early retirement. Duval (2003) examines the impact of old-age pension systems and other social transfer programmes on the retirement decision of older males in 22 OECD countries, using panel data econometrics. The main variable of interest is the implicit tax rate on continued work, which accounts for the change in net pension wealth from working an additional year ('additional' benefits and additional contributions, taken as a share of earnings). The dependent variable is the difference in male labour force participation rates between two consecutive age groups. In line with other panel data macroeconometric evidence, such as Blöndal and Scarpetta (1998) and Johnson (2000), he finds lower effects of these incentive measures than microeconometric studies, but the estimate is still highly significant. In his preferred specification, a ten percentage point decline of the implicit tax rate reduces the decline in participation rates between two consecutive age groups by about 1.5 percentage points. Based on simulations in the Gruber and Wise study, he claims that his results are two to three times lower than elasticities found there. Börsch-Supan (2000) estimates that a decrease in benefits by 12 percent would reduce the retirement probability of the 60 years-old from 39.3 percent to 28.1 percent. This amounts to a semi-elasticity of retirement with respect to benefits equal to 0.93. However, this value decreases with age. For 64 year-olds, it is estimated to be 0.45. These estimates would give a higher sensitivity of the retirement decision than the other two elasticities cited here. Based on these three estimates, we calibrate the retirement margin so that the participation rate of older workers increase by 1.7 percentage points if the participation tax rates of older workers decrease by 10 percentage points (on average of the countries modelled in PuMA).

Search Elasticity The calibration of the search elasticity is based on the impact of labour market policies on labour market outcome. In particular, estimates of the effects of an increase of the unemployment replacement rate are of special interest. We set a slightly lower elasticity than the average value stated in Nickell et al. (2005), implying that a 10 percentage point rise of the replacement ratio increases the unemployment rate by 1.11 percentage points. We calibrate the reactivity of the search intensity so that a value of 1 percentage point is an outcome of a general equilibrium simulation (on average of the countries modelled), if higher social expenditure (higher benefits and higher unemployment rate) and lower revenues (due to lower economic activity) are financed by an increase of labour taxes.

11.3 Production Side

11.3.1 Production Function

The different types of labour input are imperfect substitutes. We follow empirical evidence and assume that high-skilled labour and capital are more complementary than unskilled labour and capital ('capital-skill complementarity'). First empirical evidence is presented in Griliches (1969) who finds that capital and skilled labour inputs are more complementary than capital and unskilled labour. Duffy et al. (2004) find only weak empirical support for capital-skill complementarity, but their findings favour a more general two-level CES form over a restricted CES-nested-in-Cobb-Douglas specification. We therefore use a three-level CES form in PuMA. They also find evidence in support of complementarity, once the defined threshold for skill disjunction is low. Following this result we also set the threshold for low-skilled workers low with ISCED levels 0-2. Goldin and Katz (1998) argue that capital-skill complementarity may be a transitory phenomenon. In contrast to that, Krusell et al. (2000) find that capital-skill complementarity and changes in inputs in the aggregate production function can explain most of the increase of the skill premium observed in many countries.

The model uses a three-step nested CES production function with 3 types of labour and capital as input. Such an elaborate production function implies that several parameters have to be set. As shown later, it is necessary to set values for the substitution elasticities and the share parameters. If the production function has more than two inputs, there are several possible definitions of substitution elasticities. We restrict ourselves to the Allen-Uzawa partial elasticity of substitution as in Jaag (2005). Let σ_i denote the elasticity of substitution between capital and labour input of skill type i (1: low-skilled individuals, 2: medium, 3: high). Capital-skill complementarity implies that

$$\sigma_i < \sigma_j \qquad \text{for } i > j.$$

Krusell et al. (2000) estimate two elasticities of substitution between labour and equipment capital stock. They estimate the elasticity between unskilled (resp. skilled) labour and capital being equal to 1.67 (resp. 0.67). These values are used in Lindquist (2004) and Jaag (2005). Therefore, our elasticities are based on these estimate. However, as we use three skill groups in the model it is necessary to adjust these elasticities.

In addition to empirical estimates of elasticities of substitution, we calibrate the parameter values in a way that ensures a reasonable elasticity of physical investment on investment incentives. Empirical research on this issue often estimates an elasticity of long-run investment w.r.t. the user costs of capital (UC), which determine the necessary pre-tax return of investment. The magnitude of the response of long-run investment to UC, which combine the interest, the depreciation, the corporate tax rate and further fiscal policy parameters, is essential for the effects of policy changes on the economy. A reduction of UC, caused e.g. by a cut of the corporate tax rate, should increase investment by firms. The definition of UC can be found in the model documentation. In a review of the literature, Hassett and Hubbard (2002) argue that recent empirical analysis appears to have reached a consensus that the elasticity ranges between -0.5 and -1. Based on firm-level data for Germany, Harhoff and Ramb (2001) estimate an elasticity of -0.42. Based on a micro dataset containing around 26.000 observations, Chirinko et al. (1999) find an even lower elasticity of -0.24. We calibrate the parameters of the production function so that we find a reasonable reaction of investment to investment incentives. The corresponding values for the Allen-Uzawa elasticities which give an adequate reaction of investment to a change of UC are 1.67, 0.8 and 0.1 for low-, medium-, and high-skilled labour input. The labour share parameters are set in such a way that income shares implied by the production function correspond to the derived values within the calibration (see section 11.1.1).

Box: Derivation of Elasticities We take Allen-Uzawa elasticities from empirical literature as just described. Given these elasticities, one can derive the substitution elasticities in the production function. Therefore one needs to derive a relationship between the Allen-Uzawa elasticities and the substitution elasticities in the production function. Use the production function and define:

$$Y = F^Y(K, L_1^D, L_2^D, L_3^D) = A \cdot z_1, \tag{111}$$

where

$$\begin{aligned} z_1 &= \left[a_1 \left(L_1^D\right)^{\pi_1} + (1-a_1)\, z_2^{\pi_1}\right]^{1/\pi_1}; \\ z_2 &= \left[a_2 \left(L_2^D\right)^{\pi_2} + (1-a_2)\, z_3^{\pi_2}\right]^{1/\pi_2}; \\ z_3 &= \left[a_3 \left(L_3^D\right)^{\pi_3} + (1-a_3)\, K^{\pi_3}\right]^{1/\pi_3}. \end{aligned}$$

Capital utilization u is set to 1 in the initial steady state. In this case it can be neglected in the following analysis. One can derive the unit factor and composite demand by unit cost minimization as well as composite costs:

$$\begin{aligned} F_i &= z_i \left(\frac{a_i c_i}{w_i}\right)^{\sigma_i}, \\ c_i &= \left[(a_i)^{\sigma_i} (w_i)^{1-\sigma_i} + (1-a_i)^{\sigma_i} (c_{i+1})^{1-\sigma_i}\right]^{\frac{1}{1-\sigma_i}}, \\ z_{i+1} &= z_i \left[\frac{(1-a_i)\, c_i}{c_{i+1}}\right]^{\sigma_i}, \end{aligned}$$

where $\sigma_i \equiv \frac{1}{1-\pi_i}$. Hence, unit factor and composite demands are:

$$\begin{aligned} L_1^D &= z_1 \left(\frac{a_1 c_1}{w_1}\right)^{\sigma_1}, \quad L_2^D = z_2 \left(\frac{a_2 c_2}{w_2}\right)^{\sigma_2}, \\ L_3^D &= z_3 \left(\frac{a_3 c_3}{w_3}\right)^{\sigma_3}, \quad K = z_3 \left(\frac{(1-a_3)\, c_3}{w_k}\right)^{\sigma_3}, \\ z_2 &= z_1 \left[\frac{(1-a_1)\, c_1}{c_2}\right]^{\sigma_1}, \quad z_3 = z_2 \left[\frac{(1-a_2)\, c_2}{c_3}\right]^{\sigma_2}, \end{aligned}$$

and composite costs are given by:

$$\begin{aligned} c_1 &= \left[(a_1)^{\sigma_1} (w_1)^{1-\sigma_1} + (1-a_1)^{\sigma_1} (c_2)^{1-\sigma_1}\right]^{\frac{1}{1-\sigma_1}}, \\ c_2 &= \left[(a_2)^{\sigma_2} (w_2)^{1-\sigma_2} + (1-a_2)^{\sigma_2} (c_3)^{1-\sigma_2}\right]^{\frac{1}{1-\sigma_2}}, \\ c_3 &= \left[(a_3)^{\sigma_3} (w_3)^{1-\sigma_3} + (1-a_3)^{\sigma_3} (w_k)^{1-\sigma_3}\right]^{\frac{1}{1-\sigma_3}}. \end{aligned}$$

The cross-price elasticities between labour and capital are calculated in the following way. By forward substitution, we derive total demand for labour input of group i and calculate cross price elasticities between labour input L_i^D and K as:

$$\begin{aligned} \frac{\partial L_1^D}{\partial w_k}\frac{w_k}{L_1^D} &= \sigma_1 \frac{w_k K}{c_1 z_1}, \\ \frac{\partial L_2^D}{\partial w_k}\frac{w_k}{L_2} &= \left[\sigma_2 - \sigma_1 + \sigma_1 \frac{c_2 z_2}{c_1 z_1}\right] \frac{w_k K}{c_2 z_2}, \\ \frac{\partial L_3^D}{\partial w_k}\frac{w_k}{L_3^D} &= \left[\sigma_3 - \sigma_2 + (\sigma_2 - \sigma_1)\frac{c_3 z_3}{c_2 z_2} + \sigma_1 \frac{c_3 z_3}{c_1 z_1}\right] \frac{w_k K}{c_3 z_3} \end{aligned}$$

The Allen-Uzawa elasticity of substitution between capital and the three types of labour, which is defined as $\sigma^{L_i,K} = \frac{\partial L_i^D}{\partial w_k}\frac{w_k}{L_i^D}\frac{c_1 z_1}{w_k K}$, is thus given by:

$$\begin{aligned}
\sigma^{L_1,K} &= \sigma_1, \\
\sigma^{L_2,K} &= (\sigma_2 - \sigma_1)\frac{c_1 z_1}{c_2 z_2} + \sigma_1, \\
\sigma^{L_3,K} &= (\sigma_3 - \sigma_2)\frac{c_1 z_1}{c_3 z_3} + (\sigma_2 - \sigma_1)\frac{c_1 z_1}{c_2 z_2} + \sigma_1.
\end{aligned}$$

More general (for more input factors), Allen-Uzawa elasticities are given by

$$\sigma^{i,K} = \sigma_1 + c_1 z_1 \sum_{n=2}^{i} \frac{(\sigma_n - \sigma_{n-1})}{c_n z_n}.$$

Therefore,

$$\begin{aligned}
\sigma_1 &= \sigma^{L_1,K}, \\
\sigma_2 &= \frac{c_2 z_2}{c_1 z_1}\left(\sigma^{L_2,K} - \sigma_1\right) + \sigma_1, \\
\sigma_3 &= \frac{c_3 z_3}{c_1 z_1}\left(\sigma^{L_3,K} - \sigma_1\right) - \frac{c_3 z_3}{c_2 z_2}(\sigma_2 - \sigma_1) + \sigma_2.
\end{aligned}$$

To derive the share parameters a_1, a_2 and a_3 we start by dividing the marginal product of capital by the marginal product of the third skill group. This gives:

$$\frac{F_K^Y}{F_{L_3}^Y} = \frac{(1 - a_3) K^{\pi_3 - 1}}{a_3 \left(L_3^D\right)^{\pi_3 - 1}},$$

and:

$$a_3 = \frac{F_{L_3}^Y K^{\pi_3 - 1}}{F_{L_3}^Y K^{\pi_3 - 1} + F_K^Y \left(L_3^D\right)^{\pi_3 - 1}} = \frac{1}{1 + \frac{F_K^Y K}{F_{L_3}^Y L_3^D}\left(\frac{L_3^D}{K}\right)^{\pi_3}}.$$

Knowing the income shares of capital and of the different skill groups and the parameter π_3, and we already know L_3^D and K, we can calibrate a_3. z_3 is then simply given by the functional form (111). For a_2 and a_1 this implies:

$$\begin{aligned}
a_2 &= \frac{1}{1 + \frac{F_{z_3}^Y z_3}{F_{L_2}^Y L_2^D}\left(\frac{L_2^D}{z_3}\right)^{\pi_2}}; \\
a_1 &= \frac{1}{1 + \frac{F_{z_2}^Y z_2}{F_{L_1}^Y L_1^D}\left(\frac{L_1^D}{z_2}\right)^{\pi_1}}.
\end{aligned}$$

For the calculation of income shares for the different labour inputs, low-, medium-, and high-skilled labour and the capital share, see section 11.1.1.

11.3.2 Vacancy Costs

The most common way of modelling firm's costs for posting vacancies is to assume that they are linear in the number of vacancies. We allow for non-linear vacancy costs as it gives us additional freedom in getting economic sensitivities of policy reforms that are in line with empirical literature. Rotemberg (2006) argues that vacancy costs might be concave for several reasons and uses a value that would translate to $\varepsilon = -1/0.8$ for our vacancy cost function (see Section 10). On the other hand, there are several authors who estimate or use convex vacancy costs. Krause and Lubik (2008) use Bayesian estimation methods for data from the Haver Analytics database and estimate a mean of $\varepsilon = 4$. Bertola and Garibaldi (2001) use a quadratic cost function, so that $\varepsilon = 1$. Yashiv (2006) and Thomas (2006) cite Merz and Yashiv (2007) for a cubic function of hiring costs, $\varepsilon = 0.5$. We follow the latter and assume a value of $\varepsilon = 0.5$.

11.3.3 Capital Adjustment Costs

Investment in the model is associated with adjustment costs (see Barro and Sala-i-Martin (1995)), which ensure that firms adjust their capital stock smoothly to the optimal level in response to a policy shock. We set the parameter of the adjustment cost function such that it gives a reasonable dynamic adjustment. Radulescu and Stimmelmayr (2007) analyse the switch to an Allowance for Corporate Equity (ACE) or to a Comprehensive Business Income Tax (CBIT) type of tax system starting from the present German tax system. They set the parameter of the adjustment cost function so that half of the long-run increase in the capital stock is accumulated during the 8 years following the policy shock. In a description of the MIRAGE model, Decreux and Valin (2007) set the parameter so that half of the adjustment of the capital stock towards the long run is made in 4 years. Accordingly, we choose a period of 6 years, the average of the two values cited.

11.3.4 Layoff Rate and Employment Protection

The calibration of the firing probability is based on the share of quits and layoffs according to data from LFS and EU-SILC. Given this share it is possible to derive the layoff probability in the model. The number of quits in an age- and skill-group is $(1 - \varepsilon_i^a - u_i^a)\,\delta_i^a \bar{\delta}_i^a N_i^a$. The number of layoffs in the economy in each age- and skill-group is given by $\left(1 - p_{man,i}^a\right)(\varepsilon_i^a + (1 - \varepsilon_i^a)\, s_i^a f_i^a)\,\delta_i^a \bar{\delta}_i^a N_i^a$. The ratio of quits to layoffs is therefore given by:

$$\frac{\text{quits}}{\text{layoffs}} = \frac{(1 - \varepsilon_i^a - u_i^a)\,\delta_i^a \bar{\delta}_i^a N_i^a}{\left(1 - p_{man,i}^a\right)(\varepsilon_i^a + (1 - \varepsilon_i^a)\, s_i^a f_i^a)\,\delta_i^a \bar{\delta}_i^a N_i^a} = \frac{(1 - \varepsilon_i^a - u_i^a)}{\left(1 - p_{man,i}^a\right)(\varepsilon_i^a + (1 - \varepsilon_i^a)\, s_i^a f_i^a)}$$

Using $\varepsilon_i^a + (1 - \varepsilon_i^a)\, s_i^a f_i^a = hir_i^a = \frac{(1-u_i^a)}{p_{man,i}^a}$ gives:

$$\frac{\text{quits}}{\text{layoffs}} = \frac{1 - \varepsilon_i^a - u_i^a}{\frac{(1-p_{man,i}^a)}{p_{man,i}^a}(1 - u_i^a)}$$

The aggregate layoff rate in the the model is given by:

$$\text{layoff rate} = \frac{\sum_{i,a}\left(1 - p_{man,i}^a\right)(\varepsilon_i^a + (1 - \varepsilon_i^a)\, s_i^a f_i^a)\,\delta_i^a \bar{\delta}_i^a N_i^a}{\sum_{i,a} p_{man,i}^a\,(\varepsilon_i^a + (1 - \varepsilon_i^a)\, s_i^a f_i^a)\,\delta_i^a \bar{\delta}_i^a N_i^a}$$

A literature review on both theoretical and empirical research on EPL can be found in the first part of 'Modelling of Labour Markets in the European Union' (Berger et al. (2009)). In the model, the elasticity of the layoff rate w.r.t. EPL is based on estimates in OECD (2004). Based on a cross-country GLS estimation, the authors find that the flow into unemployment decreases by 0.165 percentage points if the OECD EPL index increases by 1 point. This result is used for the calibration of the sensitivity of the layoff decision of firms.

We implement the relative strictness of EPL by calculating a Modified EPL Index for PuMA by using version 3 of the OECD EPL Index[38] and by weighing the sub-indices for regular and temporary workers by their respective share in the labour market (taken from Eurostat). PuMA includes both severance payments and administrative firing costs. We calculate the share of severance payments on total firing costs by classifying the costs for the different items of the EPL index according to whether they are associated with severance payments or with administrative costs.

As we have more detailed information on firing costs for Germany based on Grund (2003) and Goerke and Pannenberg (2005), the calibration of firing costs in all countries in PuMA is implemented relative to these values according to their relative modified EPL index. Using these estimates and the average tenure for each age- and skill-group from the LFS, we derive average severance payments for the different groups. Thereafter, we derive administrative costs as a multiple of severance payments as defined above. In our opinion, this method is a plausible approximation for firing costs in a country.

11.4 Goods Demand and Demand Elasticities

Monopolistic competition allows to consider the impact of demand for domestic goods on the economy. The model distinguishes demand for domestic private and public consumption, investment demand as well as export demand. One has to keep in mind that the model reconsiders only domestic value added in domestic goods.For this reason foreign intermediary goods are not taken into account. This has an important impact on the value of exports and imports. Both differ from the values of the System of National Accounts due to the abstraction away from intermediary goods. Therefore exports and imports in the model do not match the corresponding variables in the National Accounts.

For the calibration of the model it is necessary to define the share of domestically- and foreign-produced goods for the different demand categories. These shares are based on the Input-Output tables provided by Eurostat. Public consumption exhibits the highest share of domestically produced value added. In contrast, investment goods are to a large extent imported goods.

With respect to elasticities, we follow the results of Ratto et al. (2009). They estimate demand elasticities for the Quest III model which we also apply in the PuMA model. According to these results, the elasticity of substitution between bundles of domestic and foreign goods for investment and public and private consumption is set to 1.17 and the Armington elasticity to 2.53. The elasticity of substitution between different varieties of goods, σ, defines the mark-up of prices over marginal costs. Setting $\sigma = 11$ imlies a mark-up of 10 percent, following Ratto et al. (2008).

[38] Compared to version 2, version 3 (which is available from 2008) comprises three additional items related to employment protection.

11.5 Labour Market

11.5.1 Matching Function

Various studies report different empirical estimates of the matching elasticity in the matching function. This may result from different ways of measuring the search intensity. Some authors use the unemployment rate as a proxy. Furthermore, it is often not clear what the term 'matching' means. Over the business cycle, the share of flows from unemployment to employment changes compared to the job to job flow. In addition, matching does not only consist of persons changing from unemployment to employment but also from out-of-labour-force to employment. Furthermore, the concentration on unemployment neglects the possibility that persons can leave the labour force, which has an impact on the unemployment rate. Problems of measuring the proper values are discussed in Broersma and van Ours (1998).

The elasticities for vacancies and 'unemployment' differ even when the authors find that the matching function is linearly homogeneous. Broersma and van Ours (1998) show that if the stock of non-unemployed persons is ignored, the true value of the matching elasticity with respect to the job seekers is underestimated, if hires are used as an indicator for matches. Additionally, if the flows from unemployment to job are used as indicator for matches the true value is overestimated. They also provide a good survey about empirical results, confirming their thesis. If unemployment outflow is used as indicator for matches, the elasticity lies between 0.6 and 0.8 in their study sample. If the total outflow into a job is used the elasticity lies between 0.3 and 0.4[39]. The results of their own estimates depend on the assumption of non-unemployed workers changing their job, i.e. being matched. In one specification, in which they also cannot reject constant returns to scale with respect to search intensity and vacancies, the elasticity of unemployed persons in the matching function is 0.74. Their survey also shows that, depending on the specification, the elasticities take similar values in different countries.

Petronglo and Pissarides (2001) also discuss the results of estimations of the matching elasticity. They find that most of these studies cannot reject a linear homogeneous matching function. The results for the elasticity of unemployment in the estimates vary significantly. Several estimates center around 0.7 and others around 0.5.

Hynninen, Kangasharju, and Pehkonen (2006) use Finnish data of the Local Labour Force Offices. As dependent variable they use the outflow from unemployment to employment during a month, excluding outflow to subsidized jobs. In their estimation, the elasticity of hirings with respect to unemployed job seekers lies between 0.74 and 0.79 for different specifications and between 0.09 and 0.11 for vacancies. Albaek and Hansen (2004) use a VAR-approach to estimate the impact of unemployment and vacancies on matching. The dependent variable is the hirings during a period. In their estimation the coefficient on unemployment is 0.74. When allowing for a deterministic change of the intercept over time, the estimation decreases to 0.66. Additionally they include two turbulence measures in the estimation (replacement ratio and industry turbulence) separately. Accounting for this decreases the elasticity further to 0.54 (in case of the replacement ratio) and 0.61 (in case of industry turbulence).

[39]One study for Germany shows an elasticity of 0. All other studies are within this range.

The parameter of the matching function is very often closely related to the bargaining power (see section 11.5.2) since it can be shown that the unemployment rate is at a socially efficient value if the bargaining power is the same as the parameter of the matching function in a standard matching model ('Hosios (1990) condition'). If the bargaining power of workers is higher than their share in the matching function, then the equilibrium unemployment is inefficiently high, creating first order welfare gains from reducing the unemployment rate. Keuschnigg (2005) shows, that the Hosios condition is also applicable in a static version of the matching model.

In many applications the matching elasticity with respect to the unemployed is set to 0.5 or close to it. Blanchard and Diamond (1989), Andolfatto (1996), Farmer and Hollenhorst (2006), and Merz (1995) set it to 0.4, Mortensen and Nagypal (2007) to 0.46 and Farmer (2005) to 0.5. Hall (2005) and Shimer (2005) diverge from this as they set the elasticity to 0.24 and 0.72. This overview can be found in Gertler and Trigari (2005), who set a value of 0.5.

Burgess and Turon (2005) and van Ours (2007) use a matching elasticity of 0.5 with respect to vacancies and set the bargaining power also to 0.5 to fulfil the Hosios condition. They not only include unemployed persons in the matching function, but also persons who perform on-the-job-search. Krause and Lubik (2007) set the matching elasticity with respect to unemployed and on-the-job searching individuals to 0.4 referring to Petronglo and Pissarides (2001). They set the bargaining power to 0.5. Oskamp and Snower (2007) set the bargaining power to 0.245 arguing that the bargaining power of workers decreased in the last years. In the MIMIC-model the authors set the bargaining power of workers to 0.03, the matching elasticity to 0.5, see Bovenberg et al. (2000). Gertler and Trigari (2006) set the bargaining power to 0.5.

In our model a deliberate estimate of the matching efficiency parameter ($m^a_{0,i}$, see the functional form) is not necessary. The model is calibrated in such a way that a higher calibrated efficiency parameter leads to a decrease of the calibrated vacancies. A change of the efficiency parameter in the calibration does not change any simulation results. Burgess and Turon (2005) set this parameter to 0.6, Cardullo and van der Linden (2006) to 0.5, and Christoffel and Kuester (2008) to 0.4.

We follow closely the Hosios condition and the parameter of the bargaining power of 0.82 for workers (see 11.5.2), we set the matching elasticity to 0.8 which means that the unemployment rate is slightly above the socially optimal level.

11.5.2 Bargaining Power

When calibrating the bargaining power in the Nash-bargaining environment, we have to be aware of the fact, that the bargaining power in our static matching model has a different interpretation than the bargaining power in a dynamic version. The crucial difference to a dynamic matching model is the outside option of workers. If, for example, there are no unemployment benefits and no value of home production, the outside option of a worker is zero in a static bargaining model. In a dynamic Mortensen-Pissarides model, the outside option is different from zero as it also takes into account the chance to get a job in the future. The outside option of firms is zero in both the static and the dynamic bargaining versions (because of the free entry condition in the dynamic version).

For the calibration of the model, we compute the bargaining power in a simple static model that is necessary to reproduce the wage that would result with a bargaining power of 0.5 in the analogous dynamic version. This simple model includes firm's and worker's contributions t^F and t^W and an unemployment benefit z, that is partly indexed to wages w and has a flat component z_0 ($z = b \cdot w + z_0$). The productivity of a worker is given by y.

Static Version In the static version, the surpluses of the worker and the firm are given by

$$\begin{aligned} S^W &= \left(1 - t^W\right) w - z; \\ S^F &= y - \left(1 + t^F\right) w. \end{aligned}$$

The wage follows from the maximization of the Nash-product,

$$\arg\max_w \left(S^F\right)^{\xi} \left(S^W\right)^{1-\xi},$$

where ξ is the bargaining power of the firm, hence

$$w = \frac{(1-\xi)\, y \left(1 - t^W - b\right) + \xi \left(1 + t^F\right) z_0}{(1 + t^F)\,(1 - t^W - b)}. \tag{112}$$

Dynamic Version The derivation of the wage in the dynamic version is slightly more difficult. We only provide results, for a more complete description of the standard Mortensen-Pissarides model, see e.g. Cahuc and Zylberberg (2004). Nash-bargaining implies that the wage is given by

$$\arg\max_w \left(\Pi_e - \Pi_v\right)^{\xi} \left(V_e - V_u\right)^{1-\xi}.$$

The expected profits from filled and vacant jobs are given by

$$\begin{aligned} r\Pi_e &= y - \left(1 + t^F\right) w + s\left(\Pi_v - \Pi_e\right); \\ r\Pi_v &= -h + m\left(\Pi_e - \Pi_v\right), \end{aligned}$$

where r is the discount rate, s is the (exogenous) probability that a filled job is separated, h are the costs of a vacant job and m is the probability of filling a vacant job. On the household side, the values of a having a job and being unemployed are given by

$$\begin{aligned} rV_e &= \left(1 - t^W\right) w + s\left(V_u - V_e\right); \\ rV_u &= z + \theta m\left(V_e - V_u\right), \end{aligned}$$

where $\theta \cdot m$ is the probability of finding a job. In this model, the wage is given by

$$\begin{aligned} w &= \frac{(1-\xi)\, y \left(1 - t^W - b\right) + \xi \left(1 + t^F\right) rV_u}{(1 + t^F)\,(1 - t^W) - (1 - \xi)\, b\,(1 + t^F)} = \\ &= \frac{(1-\xi)\, y \left(1 - t^W - b\right) + \xi \left(1 + t^F\right) \left(z_0 + \theta m\left(V_e - V_u\right)\right)}{(1 + t^F)\,(1 - t^W - b)}. \end{aligned} \tag{113}$$

By comparing (112) and (113), it becomes clear that the difference is caused by the possibility of getting a job in the future. Thus, a certain bargaining power would result in much higher wages in a dynamic model than the same bargaining power in a static model.

Calibration Based on these equations, we look for the bargaining power in the static model that gives the same wage (and the same responsiveness of the wage to changes of institutional parameters) as a bargaining power of 0.5 in the dynamic model, the value that is most often used in the literature.[40] The 'static value' is invariant with respect to the institutional parameters, i. e. the two tax rates and the calculation of the unemployment benefit. It depends on the three variables s, θm and r, but it is not very sensitive to these parameters. For reasonable parameters ($r = 0.03$, $s = 0.2$ and $f = 0.85$), the resulting bargaining power of the firm is given by 0.18, which we use in the model. The bargaining power is very often closely related to the parameter of the matching function (see section 11.5.1 on the Hosios condition).

11.6 Human Capital Production

11.6.1 Discrete Skill Choice

As described in the model documentation, individuals endogenously decide on the optimal educational level. This is done by comparing the indirect utility of two different states of education and the incremental effort that is necessary to reach the next highest level of education. Given the heterogeneity of agents w.r.t. the effort costs of reaching higher education, individuals are split into three different educational levels.

Several studies deal with the impact of tuition fees or subsidies to college enrolment in the United States and they argue that the impact may be substantial. There are only a few studies providing evidence with respect to other factors. In general, the effect of subsidies or tuition fees may differ from other influencing factors like the expected rate of return of education. If subsidies attenuate credit constraints then the effect of subsidies on enrolment may be much stronger than the effect of an increase of the rate of return.

Kane (1994) analyses the role of college costs, family background, and the rates of return on the change in attendance. He simulates the effect of a $1,000 increase of tuition fees and finds that enrolment rates in the lowest family income quartile decline by 8.5 for blacks and by 4.6 percentage points for whites. Furthermore, he tries to explain the strong change of high-school graduation in the 1970s and 1980s and finds that a $100 increase in average weekly wages in manufacturing leads to a three-percentage-point decline in high-school graduation.

Fredriksson (1997) estimates the impact of labour market variables (i.e. the wage and unemployment rates), and grants on the odds ratio of enrolment into university. The dependent variable is either the odds ratio of a steady-state approximation of the enrolment rate of the new entrants or of the share of students based on the LFS. An increase of the after tax wage of senior high-school graduates by 1 percent decreases enrolment by between 0.15 and 0.25 percentage points. An increase of net wages of university graduates by 1 percent increases enrolment by 0.35 to 0.55 percentage points. An increase of

[40] See section 11.5.1.

the unemployment rate of white collar workers by 1 percent decreases enrolment by about 0.05 percentage points.

Heckman, Lochner and Taber (1999) simulate the long-run effect of an increase of $5,000 (in 1995 Dollars) tuition subsidy on enrolment into college. The tuition subsidy corresponds to about 6 percent of the present value of earnings of a person attending college. In the simulation of Heckman, Lochner and Taber the share of young persons enrolling into college increases by 2.3 percentage points.

Winter-Ebmer and Wirz (2002) try to measure the impact of public education expenditures on enrolment into higher education, controlling for different other influences, i.e. the rate of return, the unemployment rate, entry exams and tuition fees. They use data from 14 European countries for the period 1980 to 1996. Tuition fees and entry exam variables are constructed as indicator variables. They estimate different OLS-specifications (including country and time dummies) and an instrumental variable specification. The authors find that a 1 percent increase in public funding of overall education increases enrolment into higher education by at most 1 percent. Additional public funding of higher education instead has no additional effect.

Belot, Canton and Webbink (2004) analyse the effect on enrolment of a reform of the grant system in the Netherlands implemented in the year 1996. The reform reduced the duration of student support on average by about one year, leading to a decline in grants of 4,385 euro. They find that enrolment to university fell by 2.2 percent, and find an even stronger effect of 4.7 percent on talented students. However, the probability of dropping out of university after five months declined by 2 percent.

We calibrate the elasticity of the educational decision by using estimates of Steiner and Wrohlich (2008) for Germany and Nielsen et al. (2008) for Denmark. Both studies find lower estimates than analysis for the United States. Nielsen et al. report that a USD 1,000 increase of scholarships for students in Denmark increases enrolment to university by 1.3 percentage points. Steiner and Wrohlich find that an increase of scholarships for all high-school alumni by 1,000 euro per year would increase the enrolment rate (after 5 years) to university by 2.2 percentage points (from 76.2 percent to 78.4 percent). We interpret this as a 'short-run' effect of such a reform. In the medium- and long-run, the higher number of high-skilled workers will decrease labour productivity and wages of high-skilled workers and increase unemployment relatively to the other groups. Individuals consider this for their educational decision in the general equilibrium model, so that the medium- and long-run effect is less pronounced than the short-run effect.

11.6.2 Individual Training

Given bargaining and public policy institutions, we replicate our estimates of Mincer wage equations by calibrating the age- and skill dependent labour productivity profiles.

For the accumulation of individual human capital, the model of Heckman, Lochner and Taber (1998b) can be taken as a benchmark. Based on NLSY data from 1979 to 1993, they use the following human capital production function conditional on the level of schooling S:

$$\theta^S_{a+1,t+1} = A^S \left(e^S_{at}\right)^{\alpha_S} \left(\theta^S_{a,t}\right)^{\beta_S} + \left(1-\delta^H\right)\theta^S_{a,t},$$

where $\theta^S_{a+1,t+1}$ is the stock of human capital for an individual with age $a+1$ and level of schooling S at date $t+1$, A^S is the capability (efficiency) to produce human capital, e^S_{at} is post-school investment in human capital (time) and α_S and β_S are the relative productivity of time and human capital in the production of new human capital. Based on some assumptions on skill prices and the aggregate production technology in the economy, they estimate the parameters of the human capital production function A around 0.08, $\alpha = 0.94$ and $\beta = 0.83 - 0.87$.

Fougere and Merette (1999) use the functional form

$$\theta_{t+1} = A\theta_t e_t^{\psi} + \left(1-\delta^H\right)\theta_t$$

and use $\psi = 0.7$ in their simulations. Ludwig et al. (2008) use the functional form

$$\theta_{t+1} = A\left(\theta_t e_t\right)^{\psi} + \left(1-\delta^H\right)\theta_t,$$

where e is time investment in education. They calibrate the parameters using the indirect inference method and get $\psi = 0.65$ and $A = 0.16$. In an earlier version of this model, Ludwig et al. (2007) use the functional form of Fougere and Merette and find parameters of $A = 0.08$ and $\psi = 0.8$.

For aggregation, we set $\beta = 1$ in the model. We then apply slightly lower values than Heckman, Lochner and Taber (1998) by using $A = 0.08$ and $\alpha = 0.85$ for our human capital production function. Furthermore, we allow for a negative rate of human capital depreciation. In this way, we implicitly incorporate a 'learning-by-doing' approach in the model.

11.6.3 Firm-Sponsored Training

The evidence about the effect of firm-sponsored training on wages is mixed. To mention a few, Booth and Bryan (2002) find that wage gains from participation in company training in UK are positive and persistent, whereas Goux and Maurin (2000) find statistically insignificant effects for France. Kuckulenz and Zwick (2003) report positive wage effects of participation in company training for Germany, whereas Leuven and Oosterbeck (2002) find a wage effect of training close to zero for the Netherlands. There is a growing literature that tries to assess the effect of training on productivity instead of wages. Dearden et al. (2006) find significant effects on productivity and their estimated wage effects of training are only about half of those on industrial productivity, suggesting rent-sharing between the two parties. A one percentage point increase in enrolment in training (in their dataset, around 14 percent undergo training) is associated with an increase in value added per worker of about 0.6 percent, which gives an elasticity of 0.084, and an increase in hourly wages of about 0.35 percent. Hempell (2003) investigate the relation of investments in ICT and training and their effect on productivity, using data of the German Mannheim Innovation Panel in Services. He finds an elasticity of value added w.r.t. training of between 0.117 and 0.058 (for different estimation procedures). He finds that the effect on value added is around three times as high as the elasticity of wages w.r.t. training. However, none of these values is significant (not even at a 10 percent level). Barrett and O'Connell (2001) estimate the labour productivity effects of training for enterprises in Ireland. They find a positive impact of training and elasticities of 0.099, 0.014 and 0.005 for the share of employees in the firm, the number of training days and training expenditure (training expenditure is not significant). A short review on this issue can also be found in the OECD Employment Outlook 2007 (p.64). In PuMA, the elasticity of firm-specific productivity w.r.t. the firm's investment can be derived from the functional form of the human capital formation function:

$$\varepsilon_\theta = \frac{\partial \theta^F}{\partial e^F}\frac{e^F}{\theta^F} = \alpha_{FS}\frac{1}{1+1/\left(A^F\left(e^F\right)^{\alpha_{FS}}\right)}.$$

When setting the elasticity of productivity w.r.t. firm-sponsored training, we follow Dearden et al. (2006), who estimate that an increase in enrolment in firm-sponsored training by 1 percentage point is associated with an increase in value added per worker of about 0.6 percent.

Empirical evaluation of the effects of tax incentives on training decisions of firms is very scarce. Leuven and Oosterbeek (2004) analyse the implementation of a new tax law in the Netherlands that allows for an extra tax deduction for firms that train workers above the age of 40. They find a pronounced gap of training for workers just above 40 relative to workers just below 40. Their results suggest that this difference is mainly caused by postponement of training participation. However, as their data were collected less than 2 years after the implementation, the overall effect of the new tax law is underestimated if the deduction has led to a postponement of training participation (i.e. individuals that would have been trained with 39 are now trained at 41).[41]

Bovenberg et al. (1998) set the elasticity of on-the-job training with respect to the employer's tax rate to -0.2 in their MIMIC model. We use this result as indicator for the cost elasticity for firm-specific training. Using the functional forms of the costs for firm-sponsored training and the human capital production function, the elasticity of firm-sponsored training w.r.t the marginal tax rate of the firm can be derived by totally differentiating the first order condition with respect to firm training and using the functional forms:

$$\alpha A^F\left(e^F\right)^{\alpha_{FS}-1} l_i^a \theta_i^{H,a} \underbrace{\left[p_{man,i}^a\left(F_{L,i}^Y-\left(1+t_i^{sscF,a}\right)w_i^a\right)-\left(1-p_{man,i}^a\right)\tau_i^{F,a} fac_i^a w_i^a\right]}_{k} + sub^T = fs_1\left(e^F\right)^{1/v_{FS}}$$

Totally differentiating with respect to e^F and $t_i^{sscF,a}$ gives:

$$\alpha A^F(\alpha-1)\left(e^F\right)^{\alpha-2} l_i^a\theta_i^{H,a}k\Delta e^F + \alpha A^F\left(e^F\right)^{\alpha-1} l_i^a\theta_i^{H,a}\frac{\partial k}{\partial t_i^{sscF,a}}\Delta t_i^{sscF,a} = fs_1\frac{1}{v_{FS}}\left(e^F\right)^{\frac{1}{v_{FS}}-1}\Delta e^F \Rightarrow$$

$$\frac{\Delta e^F}{\Delta t_i^{sscF,a}} = \frac{-\alpha A^F e^F l_i^a\theta_i^{H,a}\frac{\partial k}{\partial t_i^{sscF,a}}}{\alpha A^F(\alpha-1)l_i^a\theta_i^{H,a}k - fs_1\frac{1}{v_{FS}}\left(e^F\right)^{1-\alpha+1/v_{FS}}}$$

$$\text{with } \frac{\partial k}{\partial t_i^{sscF,a}} = -p_{man,i}^a w_i^a$$

[41] In a preliminary version of a different paper, Leuven and Oosterbeek (2006) find that tax incentives for households increase training participation.

The elasticity is given by multiplying this expression by $\frac{t_i^{sscF,a}}{e^F}$. To distinguish between firm-specific and non-firm-specific training, we used the LFS variables COURPURP (which indicates whether the purpose of the most recent taught learning activity was job related or not) and COURWORH (which indicates whether the most recent taught learning activity only or mostly took place within working hours, or whether it mostly or only took place outside working hours). We specify as firm specific the training that is job related and only or mostly takes place within working hours.

11.7 Institutional Details

This section gives a short overview on institutional details of the pension system, the unemployment system and other social benefits in the countries modelled. It is mainly based on the MISSOC database, OECD publications (Taxing Wages, Benefits and Wages, Pensions at a Glance) and national sources.

11.7.1 Pension System

Pension systems play a major role in providing social security in the Member States. The systems in the various countries differ significantly, both with respect to their generosity and in the breakdown between public and private pension provision and other institutional details. In many European countries, the pension system is basically a public PAYG system, but several countries also have pension benefits financed by tax revenues, a funded pillar that is often managed privately and/or occupational pension systems. Many countries are characterized by a mix between these different pillars. One should keep in mind, however, that even if a pension system is managed privately to a large extent, governments still play an important role by setting a regulatory framework or by subsidizing private saving.

The calibration of pension systems is primarily based on OECD's *Pensions at a Glance* (2015), which provides information on country-specific settings for the year 2014, the *MISSOC database* of the European Commission and *national sources*. Many countries have reformed their systems in recent years, often intended to improve the sustainability of pension systems. These reforms are usually accompanied by transition periods between the 'old' and the 'new' system. As a basic rule, we model the new system which is in place after the period of transition has ended. In our view, this approach ensures an adequate illustration of labour market incentives for those individuals currently participating on the labour market. In order to reflect higher current expenditures of the government and household income, we top up these pension benefits by flat (non earnings-related) pension benefits. Given some standard assumptions that we also apply in the model (such as perfect foresight and perfect capital markets), funded pension systems could be seen as perfect substitutes to private savings. Under these assumptions, private households reduce private savings one-by-one if contributions in the funded system increase. In addition, as for example shown in Keuschnigg (2005), a funded system does not distort labour market incentives under these conditions. One could therefore neglect modelling funded pension systems. Nevertheless, our approach is to include mandatory funded pension systems in the model as the government treats pensions quite differently than private savings in some of the countries, e.g. concerning the taxation of contributions, returns and benefits.

11.7.2 Unemployment System

The unemployment system is an important part of the public social system, which influences the behaviour of economic agents. According to economic theory, higher unemployment benefits imply positive incentives to participate on the labour market, but negative incentives to search for a job if a person is unemployed. Furthermore, unemployment benefits influence the wage bargaining process between workers and firms as wages in general will rise, as a consequence of higher reservation wages, if the replacement rate in the unemployment system increases. Unemployment regulations differ among countries to a wide extent. Characteristics about the unemployment system are mainly drawn from the OECD Benefits and Wages publication and the MISSOC database.

For the calibration of the model, we rely on EU-SILC data as it is not possible to translate institutional regulations one by one into the model. This is due to the fact, that institutional regulations alone do not provide information whether a person is eligible for unemployment payments and about the replacement rate as the rate often depends on the length of the unemployment spell or other important aspects. The variables which reflect the unemployment system in the model are ξ_1, b, and b^0. ξ_1 reflects the share of unemployed persons receiving benefits which depend on labour income before unemployment (earnings-related benefits). The other individuals either receive no public unemployment benefits or benefits which do not depend on labour income, like social assistance in most countries. b reflects the gross replacement rate in the public or private mandatory unemployment insurance and/or assistance system. If both unemployment insurance benefits as well as unemployment assistance benefits depend on previous labour income then b reflects both of them and the generosity depends on the shares of persons eligible for unemployment insurance and unemployment assistance (if eligibility differs between these two types, which is usually the case). In general, b and ξ_1 are based on EU-SILC data. However, if unemployment insurance is dependent on labour income but unemployment assistance is not, then more information is needed to derive ξ_1 and b. To derive the rate of eligibility and the replacement rate we refer to the variable *unemployment benefits PY090G* in the EU-SILC, where G stands for gross income. This variable contains the yearly income of a person received from several sources, like full and partial unemployment benefits, early retirement benefits, vocational training allowances, mobility or resettlement benefits, severance payments and other, but excludes family allowances. This income category is broader than required, but no better sources are available. As the income variable represents income received during the whole year we divide it by the number of months spent in unemployment represented by the variable *PL080* to calculate monthly income. To get rid of very low and very high benefits (which might, for example, be a result of high severance payments), we set very low benefits equal to zero and high benefits to an upper bound. Eligibility for unemployment compensation is derived as the number of persons with months spent in unemployment and receiving positive unemployment benefits in relation to the number of persons with months spent in unemployment. Therefore, the second type of persons includes therefore individuals with no unemployment benefit receipts. The average monthly unemployment benefit payments and the average monthly income for the different age- and skill-groups allow determining average gross replacement rates for the unemployment period. To derive monthly employee cash or near cash income we divide the income variable *PY010G* by the number of months spent in full- or part-time work (*PL073*, *PL074*).

11.7.3 Other Social Benefits

In addition to public unemployment and pension insurance, other social benefits are available for private households. The main database used for the division of benefits in different age- and skill-groups is EU-SILC. Given the availability of data the following cash transfers are reflected:

- Education allowances
- Sickness benefits
- Family allowances
- Social exclusion
- Housing allowances

Some of them are defined on an individual level (education allowances, sickness benefits), whereas the others are defined on a household level. Benefits which are only available on a household level are divided upon the household members for the calibration of the model in the following way. Each person in the household aged 25 or older and each person of a lower age whose mother and father are not members of the household, receive the same share of the total household benefit. This means that these benefits are divided equally upon this group of persons in the household.

Data of the EU-SILC about education allowances needs to be modified for the model as education is ongoing for younger age-groups. Without adjustment the share of allowances granted would be too high for low- and medium-skilled persons whereas high-skilled persons would only receive grants if they finished tertiary education. For this reason we divide education allowances for 15 to 19 years old persons according to the population share in the model between medium- and high-skilled persons. For 20 to 39 years old persons we assign all benefits to high-skilled persons. For older age-groups we use the data directly without any corrections.

Sickness benefits are assigned only to employed persons or persons receiving unemployment benefits. In addition we assume that the amount paid to employed and unemployed persons is the same. Sickness benefits are reflected in the model in the variables z_w and z_u as well as b^0, which reflect fixed transfers if a person is employed or unemployed. z_u includes sickness benefits for unemployed persons receiving income-dependent unemployment benefits, b^0 includes sickness benefits of unemployed persons receiving wage-independent benefits. Benefits for social exclusion are divided between three groups of persons, namely persons in retirement, persons not participating on the labour market[42] and persons in unemployment. This division is based on EU-SILC data by using information about how many months a person spent in

[42] Reflected by months spent disabled or/and unfit to work (*PL086*), studying (*PL087*), fulfilling domestic tasks and care responsibilities (*PL089*) or in other activity (*PL090*).

these states. Only persons spending the entire year in one of these states and with positive social exclusion benefits are considered for the model. This may distort the result to some extent but will, in our opinion, lead to a more trustworthy result than dividing income arbitrarily by counting all persons receiving social assistance. Social exclusion benefits for retired persons enter the model as lump-sum payments to private households. Benefits for inactive persons are included in z_{npar} and benefits for social exclusion for unemployed persons are included in b^0. Information about the age- and skill structure of social expenditures is based on the EU-SILC. Total expenditures for the different social expenditure categories are provided by the OECD Social Expenditure Dataset and are used to scale benefits derived from the EU-SILC. Although EU-SILC also provides information about total expenditures by aggregating individual or household data, small sample sizes may lead to an imprecise approximation of total expenditures. However, education allowances are directly taken from EU-SILC as the OECD Social Expenditure Dataset does not provide any information about this type of allowance.

12 Appendix: Documentation of the Migration Extension in PuMA

12.1 Motivation

This text complements the detailed documentation of PuMA and describes the model changes implemented in the extended version of PuMA in order to analyse migration in more detail.

The basic version of PuMA already allows for migration. However, migrants join the native group (in the respective age and skill group) such that different labour market characteristics and labour market outcomes according to (country of) origin is not feasible.

In the extended model version described here, we explicitly differentiate between native and foreign-borns, in each age and skill group. Most importantly, this change explicitly allows for separate (labour market) characteristics of natives and foreigners. This is particularly useful for migration analysis because labour market integration (in terms of e.g. participation and unemployment rates or wages) can significantly differ between natives and migrants, which influences outcomes of migration analysis.

In addition, the model extension enables us to account for imperfect substitutability of native and foreign workers. Even though this issue is discussed controversially in the literature (see e.g. the two seminal papers of Borjas (2003) and Ottaviano and Peri (2006)), it is useful to be able to allow for imperfect substitutability, for instance for sensitivity analysis.

12.2 Technical Implementation

Imperfect substitutability is implemented via incorporating the CES function

$$L^D_{j,i} = \left(a^N_{j,i}\left(L^{D,N}_{j,i}\right)^{\eta_i} + a^F_{j,i}\left(L^{D,F}_{j,i}\right)^{\eta_i}\right)^{\frac{1}{\eta_i}} \tag{114}$$

in equation (62), where N and F represent native and foreign born. The elasticity of substitution σ_i between native and foreign born workers is represented in $\eta_i = 1 - \frac{1}{\sigma_i}$. Accordingly, marginal labour productivity of native and foreign workers is given by

$$F^{Y,X}_{j,L,i} = F^{Y}_{j,L,i} a^{X}_{j,i} \left(\frac{L^{D,X}_{j,i}}{L^{D}_{j,i}} \right)^{\eta_i - 1}, \qquad X = N, F, \tag{115}$$

where $F^{Y}_{L,i}$ is determined by equation (105).

The described migration extension doubles the number of population groups in the model. However, the equations described in the main part of the present documentation remain valid, both for the native and foreign born population. Thus, we do not need to replicate the equations here. The model simply necessitates an appropriate aggregation, e.g. for public revenues and expenditures, macroeconomic aggregates and labour market outcomes.

13 Appendix: Proofs of Propositions

13.1 Expenditure Minimization Consumption and Inter-Vivo Transfers

We use the Langrangian method to solve the minimization problem (see 21)

$$\min_{c^a_{\alpha,t},tr^a_{\alpha,t}} pc^a\left(c^a_{\alpha,t}+tr^a_{\alpha,t}\right) \qquad s.t.$$
$$\left[\underbrace{\left(a^a_c\right)^{\frac{1}{\sigma_c}}\left(c^a_{\alpha,t}\right)^{\frac{\sigma_c-1}{\sigma_c}}+\left(1-a^a_c\right)^{\frac{1}{\sigma_c}}\left(tr^a_{\alpha,t}\right)^{\frac{\sigma_c-1}{\sigma_c}}}_{x}\right]^{\frac{\sigma_c}{\sigma_c-1}} \geq \bar{U}.$$

This results in the following FOC's:

$$\begin{aligned}
\frac{\partial}{\partial c^a_{\alpha,t}} &: \quad pc^a-\lambda\left[x\right]^{\frac{1}{\sigma_c-1}}\left(a^a_c\right)^{\frac{1}{\sigma_c}}\left(c^a_{\alpha,t}\right)^{-\frac{1}{\sigma_c}}=0\\
\frac{\partial}{\partial tr^a_{\alpha,t}} &: \quad pc^a-\lambda\left[x\right]^{\frac{1}{\sigma_c-1}}\left(1-a^a_c\right)^{\frac{1}{\sigma_c}}\left(tr^a_{\alpha,t}\right)^{-\frac{1}{\sigma_c}}=0\Rightarrow\\
c^a_{\alpha,t} &= \left(\frac{a^a_c}{1-a^a_c}\right)tr^a_{\alpha,t}.
\end{aligned}$$

Using the last relation for overall consumption expenditures C^a_α results in an optimal level of consumption and inter vivo transfers:

$$\begin{aligned}
pc^a c^a_{\alpha,t} &= a^a_c pc^a C^a_{\alpha,t},\\
pc^a tr^a_{\alpha,t} &= \left(1-a^a_c\right)pc^a C^a_{\alpha,t}.
\end{aligned}$$

13.2 Euler equation

We need several equations to proof the Euler equation. Starting with the definition of the term $\bar{V}^a_{\alpha,t+1}$ in 7 and simple extension results in (note that $\sigma=\frac{1}{1-\rho}$)

$$\begin{aligned}
\bar{V}^a_{\alpha,t+1} &= \omega^a V^a_{\alpha,t+1}+\left(1-\omega^a\right)V^{a+1}_{\alpha',t+1}=\\
&= \left(\omega^a C^a_{\alpha,t+1}+\left(1-\omega^a\right)C^{a+1}_{\alpha',t+1}\frac{V^{a+1}_{\alpha',t+1}/C^{a+1}_{\alpha',t+1}}{V^a_{\alpha,t+1}/C^a_{\alpha,t+1}}\left(\frac{\frac{pc^a_{t+1}}{pc^{a+1}_{t+1}}}{\frac{pc^a_{t+1}}{pc^{a+1}_{t+1}}}\right)^{\sigma}\right)\frac{V^a_{\alpha,t+1}}{C^a_{\alpha,t+1}}.
\end{aligned}$$

Inserting the definition for Λ^a finally results in

$$\bar{V}^a_{\alpha,t+1}=\left(\omega^a C^a_{\alpha,t+1}+\left(1-\omega^a\right)C^{a+1}_{\alpha',t+1}\Lambda^a_{\alpha,t+1}\left(\frac{pc^{a+1}_{t+1}}{pc^a_{t+1}}\right)^{\sigma}\right)\frac{V^a_{\alpha,t+1}}{C^a_{\alpha,t+1}}. \tag{116}$$

By dividing the FOC w.r.t. consumption and the enveloppe condition with respect to assets, we get

$$\left(C^a_{\alpha,t}\right)^{\rho-1}=\eta^a_t pc^a_t.$$

Using the definition of η_t^a and some transformation results in

$$\frac{V_{\alpha,t}^a/C_{\alpha,t}^a}{\left(pc_t^a\right)^\sigma} = \left(\frac{dV_{\alpha,t}^a}{dA_{\alpha,t}^a}\right)^\sigma. \tag{117}$$

Using this latter expression, Λ^a can also be expressed as

$$\Lambda_{\alpha,t+1}^{a+1} = \left(\frac{dV_{\alpha',t+1}^{a+1}/dA_{\alpha',t+1}^{a+1}}{dV_{\alpha,t+1}^a/dA_{\alpha,t+1}^a}\right)^\sigma. \tag{118}$$

Starting with the definition of $\bar{\eta}^a$ and using 118, we derive

$$\begin{aligned}\bar{\eta}_{t+1}^a &= \left[\omega^a \frac{dV_{\alpha,t+1}^a}{dA_{\alpha,t+1}^a} + (1-\omega^a)\frac{dV_{\alpha',t+1}^{a+1}}{dA_{\alpha',t+1}^{a+1}}\right]\left(\bar{V}_{\alpha,t+1}^a\right)^{-\frac{1}{\sigma}} = \\ &= \left[\omega^a + (1-\omega^a)\left(\Lambda_{\alpha,t+1}^a\right)^{1-\rho}\right]\frac{dV_{\alpha,t+1}^a}{dA_{\alpha,t+1}^a}\left(\bar{V}_{\alpha,t+1}^a\right)^{-\frac{1}{\sigma}}.\end{aligned}$$

Using the definition of Ω^a and 117, this results in

$$\bar{\eta}_{t+1}^a = \left(\frac{\bar{V}_{\alpha,t+1}^a}{V_{\alpha,t+1}^a/C_{\alpha,t+1}^a}\right)^{-\frac{1}{\sigma}}\frac{\Omega_{t+1}^a}{pc_{t+1}^a}. \tag{119}$$

The FOC for consumption 12 can be rewritten as:

$$C_{\alpha,t}^a\left(\beta R_{t+1}^\tau pc_t^a\right)^\sigma = G\left(\bar{\eta}_{t+1}^a\right)^{-\sigma}.$$

Inserting 119 into this equation, we get:

$$C_{\alpha,t}^a\left(\beta R_{t+1}^\tau pc_t^a \Omega_{t+1}^a\right)^\sigma = G\left(pc_{t+1}^a\right)^\sigma \frac{\bar{V}_{\alpha,t+1}^a}{V_{\alpha,t+1}^a/C_{\alpha,t+1}^a}.$$

Using 116, we derive

$$\begin{aligned}C_{\alpha,t}^a\left(\beta R_{t+1}^\tau pc_t^a\Omega_{t+1}^a\right)^\sigma &= G\left(pc_{t+1}^a\right)^\sigma\left(\omega^a C_{\alpha,t+1}^a + (1-\omega^a)\,C_{\alpha',t+1}^{a+1}\Lambda_{\alpha,t+1}^a\left(\frac{pc_{t+1}^{a+1}}{pc_{t+1}^a}\right)^\sigma\right) = \\ &= \omega^a\left(pc_{t+1}^a\right)^\sigma GC_{\alpha,t+1}^a + (1-\omega^a)\left(pc_{t+1}^{a+1}\right)^\sigma GC_{\alpha',t+1}^{a+1}\Lambda_{t+1}^a,\end{aligned}$$

which proves the Euler equation.

13.3 Law of Motion Pension Wealth

First note that we can express the term $\bar{\lambda}_{t+1}/\bar{\eta}_{t+1}$ as:

$$\begin{aligned}\frac{\bar{\lambda}^a_{t+1}}{\bar{\eta}^a_{t+1}} &= \frac{\left(\omega^a \frac{\partial V^a_{t+1}}{\partial P^a_{t+1}} + (1-\omega^a)\frac{\partial V^{a+1}_{t+1}}{\partial P^{a+1}_{2,t+1}}\right)(\bar{V}^a)^{\rho-1}}{\left(\omega^a \frac{\partial V^a_{t+1}}{\partial A^a_{t+1}} + (1-\omega^a)\frac{\partial V^{a+1}_{t+1}}{\partial A^{a+1}_{t+1}}\right)(\bar{V}^a)^{\rho-1}} = \\ &= \frac{\left(\omega^a \frac{\lambda^a_{t+1}}{\eta^a_{t+1}} + (1-\omega^a)\frac{\lambda^{a+1}_{t+1}}{\eta^{a+1}_{t+1}}\frac{\partial V^{a+1}_{t+1}/\partial A^{a+1}_{t+1}}{\partial V^a_{t+1}/\partial A^a_{t+1}}\right)\frac{\partial V^a_{t+1}}{\partial A^a_{t+1}}}{\omega^a \frac{\partial V^a_{t+1}}{\partial A^a_{t+1}} + (1-\omega^a)\frac{\partial V^{a+1}_{t+1}}{\partial A^{a+1}_{t+1}}} \\ &= \frac{\omega^a \frac{\lambda^a_{t+1}}{\eta^a_{t+1}} + (1-\omega^a)\frac{\lambda^{a+1}_{t+1}}{\eta^{a+1}_{t+1}}\frac{\partial V^{a+1}_{t+1}/\partial A^{a+1}_{t+1}}{\partial V^a_{t+1}/\partial A^a_{t+1}}}{\omega^a + (1-\omega^a)\frac{\partial V^{a+1}_{t+1}/\partial A^{a+1}_{t+1}}{\partial V^a_{t+1}/\partial A^a_{t+1}}} \\ &= \frac{\omega^a\tilde{\lambda}^a_{t+1} + (1-\omega^a)\tilde{\lambda}^{a+1}_{t+1}\left(\Lambda^a_{\alpha,t+1}\right)^{1-\rho}}{\Omega^a_{t+1}},\end{aligned}$$

where the last equation uses equation 118.

Divide the enveloppe condition for pension wealth (13b) by the enveloppe condition for assets (13a) to get

$$\frac{\lambda^a_t}{\eta^a_t} = \tilde{\lambda}^a_t = (1-\tau^{p,a})\,\nu^a_t + R^{P,a}\frac{\gamma^a\bar{\lambda}^a_{t+1}}{R^\tau_{t+1}\bar{\eta}^a_{t+1}}.$$

Using the upper expression for $\bar{\lambda}_{t+1}/\bar{\eta}_{t+1}$, we can rewrite

$$\tilde{\lambda}^a_t = (1-\tau^{p,a})\,\nu^a_t + \frac{\gamma^a R^{P,a}}{R^\tau_{t+1}\Omega^a_{t+1}}\left[\omega^a\tilde{\lambda}^a_{t+1} + (1-\omega^a)\tilde{\lambda}^{a+1}_{t+1}(\Lambda^a)^{1-\rho}\right],$$

which proves the equation of motion for the shadow price of pension wealth of retirees. Multiply by $P^a_{\alpha,t}$ to get

$$\tilde{\lambda}^a_t P^a_{\alpha,t} = (1-\tau^{p,a})\,\nu^a_t P^a_{\alpha,t} + \frac{\gamma^a R^{P,a}}{R^\tau_{t+1}\Omega^a_{t+1}}P^a_{\alpha,t}\left[\omega^a\tilde{\lambda}^a_{t+1} + (1-\omega^a)\tilde{\lambda}^{a+1}_{t+1}(\Lambda^a)^{1-\rho}\right].$$

By using the equation of motion for pension stocks of retirees (8), we can rewrite

$$\tilde{\lambda}^a_t P^a_{\alpha,t} = (1-\tau^{p,a})\,\nu^a_t P^a_{\alpha,t} + \frac{\gamma^a G}{R^\tau_{t+1}\Omega^a_{t+1}}\left[\omega^a\tilde{\lambda}^a_{t+1}P^a_{\alpha,t+1} + (1-\omega^a)\tilde{\lambda}^{a+1}_{t+1}P^{a+1}_{\alpha',t+1}(\Lambda^a)^{1-\rho}\right].$$

Thus, if we insert $S^a_{\alpha,t} = \tilde{\lambda}^a_t P^a_{\alpha,t}$, i.e. equation 24, this results in equation 20:

$$S^a_{\alpha,t} = (1-\tau^{p,a})\,P^a_{\alpha,t}\nu^a_t + \frac{\gamma^a}{R^\tau_{t+1}\Omega^a_{t+1}}G\bar{S}^a_{\alpha,t+1}.$$

13.4 Consumption function

We start by inserting the consumption function into the r.h.s. of the Euler equation:

$$\begin{aligned}C^a_{\alpha,t}\left(pc^a_t\beta R^\tau_{t+1}\Omega^a_{t+1}\right)^\sigma &= G\frac{\left(pc^a_{t+1}\right)^{\sigma-1}}{\Delta^a_{t+1}} * \\ &* \left[\begin{array}{c}\omega^a\left(A^a_{\alpha,t+1} + S^a_{\alpha,t+1} + T^a_{\alpha,t+1}\right) + \\ +(1-\omega^a)\frac{\Delta^a_{t+1}\left(pc^{a+1}_{t+1}\right)^{\sigma-1}\Lambda^a_{\alpha,t+1}}{\Delta^{a+1}_{t+1}\left(pc^a_{t+1}\right)^{\sigma-1}}\left(A^{a+1}_{\alpha',t+1} + S^{a+1}_{\alpha',t+1} + T^{a+1}_{\alpha',t+1}\right)\end{array}\right].\end{aligned}$$

From the definition of Λ and the condition $V_\alpha^a = (\Delta^a)^{1/\rho} C_\alpha^a$ for the value function, which will be shown afterwards, it can easily be shown that:

$$\frac{\Delta_{t+1}^a \left(pc_{t+1}^{a+1}\right)^{\sigma-1} \Lambda_{\alpha,t+1}^a}{\Delta_{t+1}^{a+1} \left(pc_{t+1}^a\right)^{\sigma-1}} = \left(\Lambda_{\alpha,t+1}^a\right)^{1-\rho}. \tag{120}$$

Inserting this condition, applying the definitions for $\bar{S}^a$ and $\bar{T}^a$ and noting that $A_{\alpha,t+1}^a = A_{\alpha',t+1}^{a+1}$, we can derive

$$C_{\alpha,t}^a \left(pc_t^a \beta R_{t+1}^\tau \Omega_{t+1}^a\right)^\sigma = G \frac{\left(pc_{t+1}^a\right)^{\sigma-1}}{\Delta_{t+1}^a} \left[\Omega_{t+1}^a A_{\alpha,t+1}^a + \bar{S}_{\alpha,t+1}^a + \bar{T}_{\alpha,t+1}^a\right].$$

By multiplying this equation by $\gamma^a \Delta_{t+1}^a / R_{t+1}^\tau$ and using the equations of motion for A, T and S, i.e. 10, 19 and 20, we derive:

$$\gamma^a \Delta_{t+1}^a (\beta)^\sigma \left(R_{t+1}^\tau \Omega_{t+1}^a\right)^{\sigma-1} (pc_t^a)^\sigma C_{\alpha,t}^a = \left(pc_{t+1}^a\right)^{\sigma-1} \left(A_{\alpha,t}^a + S_{\alpha,t}^a + T_{\alpha,t}^a - pc_t^a C_{\alpha,t}^a\right).$$

Inserting the optimal consumption function for $A_{\alpha,t}^a + S_{\alpha,t}^a + T_{\alpha,t}^a$ on the r.h.s. results in the equation of motion for Δ, 18. Thus the stated policy satisfies the Euler condition 15 and is optimal.

13.5 Value Function

We show that the conjecture fulfills the Bellman equation for the value function. Inserting the conjecture 17 for $C_{\alpha,t}^a$ into the Euler-equation 15 yields:

$$\frac{V_{\alpha,t}^a}{\left(\Delta_t^a\right)^{1/\rho}} \left(pc_t^a \beta R_{t+1}^\tau \Omega_{t+1}^a\right)^\sigma = G \left[\begin{array}{c} \omega^a \left(pc_{t+1}^a\right)^\sigma \frac{V_{\alpha,t+1}^a}{\left(\Delta_{t+1}^a\right)^{1/\rho}} + \\ + (1-\omega^a) \left(pc_{t+1}^{a+1}\right)^\sigma \frac{V_{\alpha',t+1}^{a+1}}{\left(\Delta_{t+1}^{a+1}\right)^{1/\rho}} \Lambda_{\alpha,t+1}^a \end{array}\right].$$

Multiplying this equation by $\left(\Delta_{t+1}^a\right)^{1/\rho}$ results in:

$$V_{\alpha,t}^a \left(\frac{\Delta_{t+1}^a}{\Delta_t^a}\right)^{\frac{1}{\rho}} \left(pc_t^a \beta R_{t+1}^\tau \Omega_{t+1}^a\right)^\sigma = G \left[\begin{array}{c} \left(pc_{t+1}^a\right)^\sigma \omega^a V_{\alpha,t+1}^a + \\ + (1-\omega^a) \left(pc_{t+1}^{a+1}\right)^\sigma V_{\alpha',t+1}^{a+1} \left(\frac{\Delta_{t+1}^a}{\Delta_{t+1}^{a+1}}\right)^{1/\rho} \Lambda_{\alpha,t+1}^a \end{array}\right].$$

Note that $\left(\frac{pc_{t+1}^{a+1}}{pc_{t+1}^a}\right)^\sigma \left(\frac{\Delta_{t+1}^a}{\Delta_{t+1}^{a+1}}\right)^{1/\rho} \Lambda_{\alpha,t+1}^a = 1$ (see 120), so that

$$V_{\alpha,t}^a \left(\frac{\Delta_{t+1}^a}{\Delta_t^a}\right)^{1/\rho} \left(pc_t^a \beta R_{t+1}^\tau \Omega_{t+1}^a\right)^\sigma = G \left(pc_{t+1}^a\right)^\sigma \left[\omega^a V_{\alpha,t+1}^a + (1-\omega^a) V_{\alpha',t+1}^{a+1}\right].$$

Taking the power of ρ and multiplying by $\gamma^a \beta$ gives:

$$\gamma^a \beta \left(V_{\alpha,t}^a\right)^\rho \frac{\Delta_{t+1}^a}{\Delta_t^a} \left(pc_t^a \beta R_{t+1}^\tau \Omega_{t+1}^a\right)^{\sigma-1} = \gamma^a \beta \left(pc_{t+1}^a\right)^{\sigma-1} \left(G \bar{V}_{\alpha,t+1}^a\right)^\rho.$$

By using dynamic equation for the marginal propensity to consume (see 18) for Δ_{t+1}^a, this equation can be simplified to:

$$\frac{\Delta_t^a - 1}{\Delta_t^a} \left(V_{\alpha,t}^a\right)^\rho = \gamma^a \beta \left(G \bar{V}_{\alpha,t+1}^a\right)^\rho.$$

Using conjecture (17) once more shows that the Bellman equation is identically fulfilled:

$$V_{\alpha,t}^a = \left[\left(C_{\alpha,t}^a\right)^\rho + \gamma^a \beta \left(G \bar{V}_{\alpha,t+1}^a\right)^\rho\right]^{1/\rho}.$$

13.6 Investment function

By using Hayashi (72) and the equation for the capital stock, we get:

$$tob_{t+1} = \frac{V_{t+1}^{K}}{K_{t+1}} = \frac{GV_{t+1}^{K}}{(1-\delta(u_t))K_t + I_t} = \frac{GV_{t+1}^{K}}{((1-\delta(u_t)) + j_t)K_t},$$

where we define $j_t = I_t/K_t$. Inserting the optimality condition w.r.t. I, (69) results in:

$$\frac{GV_{t+1}^{K}}{(1-\delta(u_t)+j)K_t} = R_{t+1}p_t^{inv}\left(1+\left(1-t^{prof}\right)J_I - t^{prof}sub^I\right).$$

Rearrangement of terms and the use of the functional form of J (i.e.: $J_I = \psi(j-\delta(u)-g)$) gives:

$$\begin{aligned}\frac{GV_{t+1}^{K}}{p_t^{inv}R_{t+1}K_t} &= \left[1+\left(1-t^{prof}\right)\psi\left(j-\delta(u_t)-g\right)-t^{prof}sub^I\right](1-\delta(u_t)+j) = \\ &= j^2\left(1-t^{prof}\right)\psi + j\left[\left(1-t^{prof}\right)\psi(1-\delta(u_t)) + 1 - t^{prof}sub^I - \left(1-t^{prof}\right)\psi(\delta(u_t)+g)\right] + \\ &\quad + (1-\delta(u_t))\left(1-t^{prof}sub^I - \left(1-t^{prof}\right)\psi(\delta(u_t)+g)\right).\end{aligned}$$

This results in the quadratic equation (for I):

$$\begin{aligned}& I^2 + I\left[(1-\delta(u_t)) - (\delta(u_t)+g) + \frac{1-t^{prof}sub^I}{\left(1-t^{prof}\right)\psi}\right]K + \\ & + \left[\left((1-\delta(u_t))\left(1-t^{prof}sub^I\right) - (1-\delta(u_t))(\delta(u_t)+g)\left(1-t^{prof}\right)\psi\right) - \frac{GV_{t+1}^{K}}{p_t^{inv}R_{t+1}K}\right]\frac{K^2}{\left(1-t^{prof}\right)\psi} \\ = \; & 0;\end{aligned}$$

which proves the proposition.

13.7 Walras' Law

We start with excess demand for assets,

$$G\xi^A = G\left(A_{t+1} - V_{t+1} - DG_{t+1} - DF_{t+1}\right),$$

and insert the equations of motion for assets, the firm value, government and foreign debt, which results in:

$$\frac{G\xi^A}{R_{t+1}} = A - V - DG - DF + W^H - pc\cdot C + \chi^K + \chi + PB - TB - \frac{t^{cg}r_{t+1}}{R_{t+1}}\left(A + W^H - pcC\right) + ex^{Pens,Corr},$$

where W^H is aggregate disposable income of individuals (consisting of wage, unemployment, pension, lump sum and inter-vivo income). Using the condition:

$$A - V - DG - DF = 0$$

and the fact that $pc\cdot C$ is split into private consumption and inter-vivo transfers given, i.e.

$$pc\cdot C = (1+t^c)c + IV,$$

and the equation for the dividends results in the equation:

$$\begin{aligned}
\frac{G\xi^A}{R_{t+1}} &= W^H - IV - pc \cdot c + \left(1 - t^{prof}\right)\left[p_t^k K_t - p_t^{inv} J_t + p_t^{inv} K_t \left(\delta\left(u_t\right) - \bar{\delta}^K\right) - t^{cap} p_t^{inv} K_t\right] + \\
&+ t^{prof}\left(\delta\left(u_t\right) p_t^{inv} K_t + sub^I p_t^{inv} I_t\right) - p_t^{inv} I_t + \\
&+ \sum_j \left(p_j \bar{Y}_j - p^k K_j - p^{inv} K_j \left(\delta\left(u_j\right) - \bar{\delta}^K\right) - W_j^F - T_j^F - firm_{tax,j}\right) \\
&+ PB - TB - \frac{t^{cg} r_{t+1}}{R_{t+1}}\left(A + W^H - pc \cdot C\right) + ex^{Pens,Corr} \\
&= \sum_j p_j \bar{Y}_j - p^{inv}\left(J + I\right) - pc \cdot c - p^{cg}\left(C^G + C^H\right) - TB + W^H - IV - t^c c - W^F - \bar{T}^F - firm_{tax} + \\
&+ PB + p^{cg}\left(C^G + C^H\right) - \frac{t^{cg} r_{t+1}}{R_{t+1}}\left(A + W^H - pc \cdot C\right) + ex^{Pens,Corr},
\end{aligned}$$

where $\bar{Y}$ is defined in the final goods optimization section and $firm_{tax}$ represents different taxes paid and subsidies received by firms that are not included in $\bar{T}^F$. $\bar{T}^F$ is given by the following expression:

$$\bar{T}^F = t^{prof}\left(\sum_j p_j \bar{Y}_j - p^{inv}\left(J + K\delta\left(u\right) + sub^I I + t^{cap} K\right) + t^{cap} p^{inv} K - W^F - firm_{tax}\right)$$

By using the definition of the excess demand for goods,

$$\xi^C = -\sum_j p_j \bar{Y}_j + p^{inv}\left(J + I\right) + \bar{pc} \cdot c + p^{cg}\left(C^G + C^H\right) + TB,$$

with $\bar{pc} = \frac{pc}{1+t^c}$ the former equation can be written as:

$$\begin{aligned}
\frac{G\xi^A}{R_{t+1}} + \xi^C &= W^H - IV - t^c \bar{pc} \cdot c - \left(1 + t^s\right) W^F - \bar{T}^F + PB + p^{cg}\left(C^G + C^H\right) - \\
&- \frac{t^{cg} r_{t+1}}{R_{t+1}}\left(A + W^H - pc \cdot C\right) + ex^{Pens,Corr} - firm_{tax}.
\end{aligned}$$

Dissolving the term W^H and noting that inter-vivo transfers received (which is included in W^H) and given (IV) cancel, we get:

$$\begin{aligned}
\frac{G\xi^A}{R_{t+1}} + \xi^C &= \sum_{j,i,a}\left(1 - \tau_i^{w,a}\right) W_{j,i}^{H,a} + \sum_{i,a} B_i^a + \sum_{i,a}\left(1 - \tau_i^{p,a}\right)\tilde{P}_i^a + \sum_{i,a} ZP_i^a + \\
&+ \sum_{i,a}\left(1 - \tau_i^{pd,a}\right) P\tilde{D}B_i^a + \sum_{i,a} ZDP_i^a + \sum_{i,a} ZW_i^a + Z - \\
&- \sum_{j,i,a}\left(1 + t_i^{s,a}\right) W_{j,i}^{F,a} - \sum_{i,a} ZFS_i^a - t^c \bar{pc} \cdot c - \bar{T}^F + ex^{Pens,Corr} - firm_{tax} \\
&+ PB + p^{cg}\left(C^G + C^H\right) - \frac{t^{cg} r_{t+1}}{R_{t+1}}\left(A + W^H - pc \cdot C\right),
\end{aligned}$$

where the respective sums are aggregates over age and skill groups and W^H and W^F are gross wages received by the households and paid by the firms. This equation can also be written as:

$$\begin{aligned}
\frac{G\xi^A}{R_{t+1}}+\xi^C &= \sum_{j,i,a}\left(W^{H,a}_{j,i}-W^{F,a}_{j,i}\right)-TW^{Work}-TSsc^{Work}+\sum_{i,a}B^a_i+\\
&+\sum_{i,a}\tilde{P}^a_i-TW^{Pens}-TSsc^{Pens}+\sum_{i,a}ZP^a_i+\\
&+\sum_{i,a}P\tilde{D}B^a_i-TW^{PDB}-TSsc^{PDB}+\sum_{i,a}ZDP^a_i+\\
&+\sum_{i,a}ZW^a_i+Z-TSsc^{Firm}+\sum_{i,a}ZFS^a_i-t^c\bar{p}c\cdot c-\bar{T}^F+ex^{Pens,Corr}-firm_{tax}+\\
&+PB+p^{cg}\left(C^G+C^H\right)-\frac{t^{cg}r_{t+1}}{R_{t+1}}\left(A+W^H-pc\cdot C\right),
\end{aligned}$$

where TW^x and $TSsc^x$ represent aggregate income taxes and social security contributions paid by workers, retirees, disabled and firms, respectively. It holds, that:

$$\begin{aligned}
\sum_{j,i,a}\left(W^{H,a}_{j,i}-W^{F,a}_{j,i}\right) &= \sum_{j,i,a}w^a_{j,i}\theta^a_{j,i}l^a_i\left(1-ue^a_i\right)\delta^a_i\bar{\delta}^a_iN^a_i-p^a_{man,j,i}\left(q^a_iv^a_{j,i}+\varepsilon^a_i\delta^a_i\bar{\delta}^a_iN^a_i\right)\theta^a_{j,i}w^a_{j,i}l^a_i=\\
&= \sum_{j,i,a}w^a_{j,i}\theta^a_{j,i}l^a_ip^a_{man,j,i}\left(\varepsilon^a_i\delta^a_i\bar{\delta}^a_iN^a_i+\left(1-\varepsilon^a_i\right)f^a_is^a_i\delta^a_i\bar{\delta}^a_iN^a_i-\varepsilon^a_i\delta^a_i\bar{\delta}^a_iN^a_i-q^a_iv^a_{j,i}\right)=\\
&= \sum_{j,i,a}w^a_{j,i}\theta^a_{j,i}l^a_ip^a_{man,j,i}\left(f^a_iS^a_i-q^a_iv^a_{j,i}\right)=\\
&= -w\xi^L.
\end{aligned}$$

Using this equation and the equations for the different public budgets, the former equation can be rewritten as:

$$\begin{aligned}
\frac{G\xi^A}{R_{t+1}}+\xi^C+w\xi^L &= Tr+p^{cg}C^G+Z+PB-rev=\\
&= ex+PB-rev=-\xi^G,
\end{aligned}$$

which proves Walras' law:

$$\frac{G\xi^A}{R_{t+1}}+\xi^C+w\xi^L+\xi^G=0.$$

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